Hindawi
Advances in High Energy Physics
Volume 2021, Article ID 4970469, 6 pages
https://doi.org/10.1155/2021/4970469

Research Article

Motion of Charged Spinning Particles in a Unified Field

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Received 15 May 2021; Accepted 11 October 2021; Published 25 November 2021

Academic Editor: Shi Hai Dong

Using a geometry wider than Riemannian one, the parameterized absolute parallelism (PAP) geometry, we derived a new curve containing two parameters. In the context of the geometrization philosophy, this new curve can be used as a trajectory of charged spinning test particle in any unified field theory constructed in the PAP space. We show that imposing certain conditions on the two parameters, the new curve can be reduced to a geodesic curve giving the motion of a scalar test particle or/and a modified geodesic giving the motion of neutral spinning test particle in a gravitational field. The new method used for derivation, the Bazanski method, shows a new feature in the new curve equation. This feature is that the equation contains the electromagnetic potential term together with the Lorentz term. We show the importance of this feature in physical applications.

1. Introduction

According to the geometrization philosophy, the curve in a certain geometry represents the equation of motion of a theory which constructed in this geometry. Together with the field equations of any theory, we need the equation of motion which characterizes the theory used. In general relativity, geodesic curve is considered as an equation of motion of a scalar test particle moving in a gravitational field.

Geodesic equation can be derived using the Lagrangian (cf. [1]):

$$L_1 \stackrel{\text{def.}}{=} g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu, \tag{1}$$

where $g_{\mu\nu}$ is the metric tensor and $\dot{x}^\mu (\stackrel{\text{def.}}{=} dx^\mu/ds)$ is the unit tangent vector to the curve. Euler-Lagrange equation is given by the following (cf. [2]):

$$\frac{d}{ds}\frac{\partial L_1}{\partial \dot{x}^\gamma} - \frac{\partial L_1}{\partial x^\gamma} = 0, \tag{2}$$

such that s is the scalar parameter varying along the curve. Using Lagrangian (Equation (1)) and Equation (2), we get the following:

$$\ddot{x}^\alpha + \left\{ \begin{matrix} \alpha \\ \mu\nu \end{matrix} \right\} \dot{x}^\mu \dot{x}^\nu = 0, \tag{3}$$

where $\left\{ \begin{matrix} \alpha \\ \mu\nu \end{matrix} \right\}$ is the coefficient of Levi-Civita linear connection which is defined as follows:

$$\left\{ \begin{matrix} \alpha \\ \mu\nu \end{matrix} \right\} \stackrel{\text{def.}}{=} \frac{1}{2} g^{\alpha\sigma} \left(g_{\mu\sigma,\nu} + g_{\nu\sigma,\mu} - g_{\mu\nu,\sigma} \right). \tag{4}$$

Equation (3) is the curve equation of the Riemannian geometry.

The Lagrangian used for deriving the equation of motion of a charged particle moving in the presence of electromagnetic field is defined by the following (cf. [3]):

$$L_2 \stackrel{\text{def.}}{=} g_{\mu\nu}(V^\mu + \beta A^\mu)V^\nu, \tag{5}$$

where A^μ is a vector field and β is a conversion parameter given by the following:

$$\beta \overset{\text{def.}}{=} \frac{e}{m}, \tag{6}$$

where e is the electron charge and m is the electron mass. Using Lagrangian (Equation (5)) and Euler-Lagrange (Equation (2)), we get the equation of motion of a charged test particle moving in a combined gravitational and electromagnetical field (cf. [4]):

$$\frac{dV^\alpha}{ds} + \begin{Bmatrix} \alpha \\ \mu\nu \end{Bmatrix} V^\mu V^\nu = -\beta F^\alpha_{.\nu} V^\nu, \tag{7}$$

where $F_{\mu\nu}$ is the curl of the vector A_μ. The term on the right hand side of Equation (7) is known as a Lorentz force term.

In 1989, Bazanski [5] suggested one Lagrangian to derive both geodesic and geodesic deviation equations in Riemannian geometry, which is given by the following:

$$L_B \overset{\text{def.}}{=} g_{\mu\nu} V^\mu \frac{D\Psi^\nu}{Ds}, \tag{8}$$

where V^μ is the unit vector tangent to the path, Ψ^ν is the deviation vector, and

$$\frac{D\Psi^\nu}{Ds} \overset{\text{def.}}{=} \Psi^\nu_{;\alpha} V^\alpha. \tag{9}$$

The semicolon operator (;) denotes covariant differentiation using Levi-Civita connection, while the comma (,) stands for ordinary partial differentiation. According to Bazanski, variation with respect to deviation vector Ψ^μ gives geodesic equation, while variation with respect to the unit vector V^μ gives geodesic deviation equation.

Riemannian geometry has a unique linear connection, which is Levi-Civita connection. Wanas et al. [6] have modified Bazanski approach in a different geometry. This geometry has other linear connections together with the Levi-Civita connection. They have applied Bazanski Lagrangian using the four linear connections defined in the absolute parallelism (AP) geometry. The four connections are Weitzenböck connection $\Gamma^\alpha_{\mu\nu}$, dual connection $\widetilde{\Gamma}^\alpha_{\mu\nu} (\overset{\text{def.}}{=} \Gamma^\alpha_{\nu\mu})$, $\Gamma^\alpha_{(\mu\nu)} (\overset{\text{def.}}{=} 1/2(\Gamma^\alpha_{\mu\nu} + \Gamma^\alpha_{\nu\mu}))$, and Levi-Civita connection. Using each connection, they have got new definitions for operator D/Ds which appears in the Bazanski Lagrangian (Equation (8)). Wanas et al. have obtained a new set of three different path equations. The three different path equations can be written as follows [6]:

$$\frac{dJ^\mu}{ds^-} + \begin{Bmatrix} \mu \\ \alpha\beta \end{Bmatrix} J^\alpha J^\beta = 0, \tag{10}$$

$$\frac{dW^\mu}{ds^0} + \begin{Bmatrix} \mu \\ \alpha\beta \end{Bmatrix} W^\alpha W^\beta = -\frac{1}{2} \Lambda_{(\alpha\beta).}^{\ \ \ \ \ \mu} W^\alpha W^\beta, \tag{11}$$

$$\frac{dV^\mu}{ds^+} + \begin{Bmatrix} \mu \\ \alpha\beta \end{Bmatrix} V^\alpha V^\beta = -\Lambda_{(\alpha\beta).}^{\ \ \ \ \ \mu} V^\alpha V^\beta, \tag{12}$$

where V^μ, W^μ, and J^μ are the unit tangent vectors to the curves characterized by parameters s^+, s^0, and s^-, respectively.

If the moving particle has another property, for example, like spin, then geodesic Equation (3) is not suitable for describing the motion of such particle. An important property of the set of Equations (10), (11), and (12) appears in its right hand side. This property is the jumping parameter of the right hand side of the abovementioned equations. This property motivates Wanas [7] to consider the right hand side of the above set of equations as representing a geometric interaction between the quantum spin of the moving particle and the torsion of the background geometry. For this property, Wanas generalized the AP space by constructing a new version called the parameterized absolute parallelism (PAP).

Parameterized absolute parallelism (PAP) geometry [7] has a spectrum of spaces. It can be reduced to Riemannian and absolute parallelism geometries in some special cases. Applying the modified Bazanski approach in the context of PAP geometry, Wanas [7] obtained a modified geodesic equation.

$$\frac{dV^\mu}{ds} + \begin{Bmatrix} \mu \\ \alpha\beta \end{Bmatrix} V^\alpha V^\beta = -b\Lambda_{(\alpha\beta).}^{\ \ \ \ \ \mu} V^\alpha V^\beta. \tag{13}$$

This equation describes the motion of a spinning particle moving in a gravitational field. This equation can reduce to geodesic one in a special case ($b = 0$).

In the framework of PAP geometry, we are going to derive the equation of motion of a spinning and charged particle moving in a combined gravitational and electromagnetic field (a unified field), using the modified Bazanski approach.

2. Geometry Used: Parameterized Absolute Parallelism Geometry

This work is carried out in the context of the "parameterized absolute parallelism" (PAP) geometry abbreviated as $(M, \underset{i}{\lambda})$ [7]. M is an n-dimensional differentiable manifold, and $\underset{i}{\lambda}$ is a set of n-independent vector fields. The components of these vector fields are considered as the building blocks (BB) (BB are geometric quantities using which we can construct all objects of the geometry.) of this geometry as it is considered in the AP geometry. Since the determinant $\lambda (= |\underset{i}{\lambda}_\mu|)$ is nonvanishing; i.e., $\lambda \neq 0$, then the covariant components $\underset{i}{\lambda}_\mu(x)$ (We use Greek indices for coordinate components written in a covariant or contravariant positions. Latin indices are used to represent vector numbers, written always in a lower position. Summation convention is carried out for Greek indices in the usual way, while for Latin dummy indices the operation is carried out wherever the indices appear in the same term.) and contravariant $\underset{i}{\lambda}^{\mu}$ components satisfy the following relations:

$$\underset{i}{\lambda}^{\alpha}\underset{i}{\lambda}_{\beta} = \delta^{\alpha}_{\beta}, \quad \underset{i}{\lambda}^{\alpha}\underset{j}{\lambda}_{\alpha} = \delta_{ij}, \tag{14}$$

where δ_{ij} is the Kronecker delta. We can define, from which, the following second-order symmetric tensors as follows:

$$g_{\mu\nu} \overset{\text{def.}}{=} \underset{i}{\lambda}_{\mu}\underset{i}{\lambda}_{\nu}, \tag{15}$$

$$g_{\mu\nu} \overset{\text{def.}}{=} \underset{i}{\lambda}^{\mu}\underset{i}{\lambda}^{\nu}. \tag{16}$$

Consequently,

$$g^{\alpha\mu}g_{\alpha\nu} = \delta^{\mu}_{\nu}. \tag{17}$$

The second-order tensor $g_{\mu\nu}$ can be used as the metric tensor to define, as a special case, a Riemannian space in the context of the PAP geometry.

The PAP linear connection is given by the following [7]:

$$\nabla^{\alpha}_{.\mu\nu} \overset{\text{def.}}{=} \left\{ \begin{matrix} \alpha \\ \mu\nu \end{matrix} \right\} + \overset{*}{\gamma}{}^{\alpha}_{.\mu\nu}, \tag{18}$$

where $\gamma^{*\alpha}_{.\mu\nu}$ is a third-order tensor, called the parameterized contortion, defined by the following:

$$\overset{*}{\gamma}{}^{\alpha}_{.\mu\nu} \overset{\text{def.}}{=} b\gamma^{\alpha}_{.\mu\nu} = b\underset{i}{\lambda}^{\alpha}\underset{i}{\lambda}_{\mu;\nu}, \tag{19}$$

such that b is a dimensionless parameter.

An important note is that inserting the parameter b will not cause any difference to the properties of the AP space, but will add to them; for example, the Riemannian geometry is defined in the AP space as an associated space to it, but in the present case, it is can be considered as a special case. This will be more clear in the text.

The parameterized connection (Equation (18)) has been proved to be a metric one [7]; i.e., it satisfies metricity condition (We use the double stroke || and (+) sign to characterize covariant differentiation using the parameterized connection (Equation (18)).)

$$g_{\underset{++}{\mu\nu\|\sigma}} \equiv 0. \tag{20}$$

Since $\nabla^{\alpha}_{.\mu\nu}$ is nonsymmetric, then the parameterized torsion tensor $\Lambda^{*\alpha}_{.\mu\nu}$ is defined by the following:

$$\overset{*}{\Lambda}{}^{\alpha}_{.\mu\nu} \overset{\text{def.}}{=} \nabla^{\alpha}_{.\mu\nu} - \nabla^{\alpha}_{.\nu\mu} = \overset{*}{\gamma}{}^{\alpha}_{.\mu\nu} - \overset{*}{\gamma}{}^{\alpha}_{.\nu\mu} = b\Lambda^{\alpha}_{.\mu\nu}, \tag{21}$$

$$\Lambda^{\alpha}_{.\mu\nu} \overset{\text{def.}}{=} \gamma^{\alpha}_{.\mu\nu} - \gamma^{\alpha}_{.\nu\mu}, \tag{22}$$

which is the torsion tensor of the AP space. The contraction of the parameterized torsion (Equation (21)) or contortion is given by the following:

$$\overset{*}{c}_{\mu} \overset{\text{def.}}{=} \overset{*}{\Lambda}{}^{\alpha}_{.\mu\alpha} = \overset{*}{\gamma}{}^{\alpha}_{.\mu\alpha} = bc_{\mu}. \tag{23}$$

Also, due to the nonsymmetry of the parameterized connection (Equation (18)), there exist two more linear connections: the dual connection defined as follows:

$$\tilde{\nabla}^{\alpha}_{.\mu\nu} \overset{\text{def.}}{=} \nabla^{\alpha}_{.\nu\mu}, \tag{24}$$

and the symmetric part of $\nabla^{\alpha}_{.\mu\nu}$ which is given as follows:

$$\nabla^{\alpha}_{.(\mu\nu)} \overset{\text{def.}}{=} \frac{1}{2}\left(\nabla^{\alpha}_{.\mu\nu} + \nabla^{\alpha}_{.\nu\mu}\right) = \left\{ \begin{matrix} \alpha \\ \mu\nu \end{matrix} \right\} + \frac{1}{2}\overset{*}{\Delta}{}^{\alpha}_{.\mu\nu}, \tag{25}$$

where

$$\overset{*}{\Delta}{}^{\alpha}_{.\mu\nu} \overset{\text{def.}}{=} \overset{*}{\gamma}{}^{\alpha}_{.\mu\nu} + \overset{*}{\gamma}{}^{\alpha}_{.\nu\mu}. \tag{26}$$

The PAP geometry has four linear connections which are $\nabla^{\alpha}_{.\mu\nu}$, $\tilde{\nabla}^{\alpha}_{.\mu\nu}$, $\nabla^{\alpha}_{.(\mu\nu)}$, and $\left\{ \begin{matrix} \alpha \\ \mu\nu \end{matrix} \right\}$. So, we have four different curvature tensors, defined by ordinary manner, using the commutation relation for each one of these connections.

For each connection, we can define the following tensor derivatives as follows:

$$K^{\overset{\alpha}{+}}_{.\|\beta} \overset{\text{def.}}{=} K^{\alpha}_{\ ,\beta} + K^{\mu}\nabla^{\alpha}_{\ \mu\beta}, \tag{27}$$

$$K^{\overset{\alpha}{-}}_{.\|\beta} \overset{\text{def.}}{=} K^{\alpha}_{\ ,\beta} + K^{\mu}\nabla^{\alpha}_{\ \beta\mu}, \tag{28}$$

$$K^{\alpha}_{.\|\beta} \overset{\text{def.}}{=} K^{\alpha}_{\ ,\beta} + K^{\mu}\nabla^{\alpha}_{\ (\mu\beta)}, \tag{29}$$

where K^{α} is any arbitrary vector field defined in the PAP space.

The curve equation characterizing PAP geometry is given by, as mentioned above, the following [7]:

$$\frac{dV^{\mu}}{d\tau} + \left\{ \begin{matrix} \mu \\ \alpha\beta \end{matrix} \right\} V^{\alpha}V^{\beta} = -b\Lambda_{(\alpha\beta)}{}^{\mu}_{.} V^{\alpha}V^{\beta}, \tag{30}$$

where

$$\Lambda_{(\alpha\beta)}{}^{\mu}_{.} \overset{\text{def.}}{=} \frac{1}{2}\left(\Lambda_{(\alpha\beta)}{}^{\mu}_{.} + \Lambda_{(\beta\alpha)}{}^{\mu}_{.}\right). \tag{31}$$

The PAP geometry reduces to the Riemannian one if we take $b = 0$, while, for $b = 1$, the PAP geometry reduces to AP geometry. At any stage of calculations, we can go back to Riemannian or AP geometries as two special cases. The dimensionless parameter b is suggested, for physical applications, to take the following value [7]:

$$b \overset{\text{def.}}{=} \frac{N}{2}\alpha\gamma, \tag{32}$$

where N is an integer number that takes the values ($N = 0, 1, 2, \cdots$), α is the fine structure constant, and γ is a dimensionless parameter to be adjusted with experiments or observations for every system.

3. Path Equation for Charged and Spinning Particles

In order to derive a general equation of motion, let us define the parameterized Lagrangian in the PAP geometry by the following:

$$L \overset{\text{def.}}{=} g_{\mu\nu}(U^\mu + \beta c^\mu)\frac{D\psi^\nu}{D\tau}, \tag{33}$$

where U^μ is the tangent to a path characterized by the parameter τ, ψ^ν is the deviation vector, and

$$\frac{D\psi^\nu}{D\tau} \overset{\text{def.}}{=} \psi^\nu_{+.\|\alpha} U^\alpha, \tag{34}$$

so, the Lagrangian (Equation (33)) can be written in the explicit form as follows:

$$L = g_{\mu\nu}(U^\mu + \beta c^\mu)U^\alpha\left(\psi^\nu_{.,\alpha} + \psi^\varepsilon \nabla^\nu_{.\varepsilon\alpha}\right). \tag{35}$$

Now, it is clear that the Lagrangian (Equation (35)) has two parameters: one is the dimensionless parameter b mentioned above in the parameterized connection $\nabla^\nu_{.\varepsilon\alpha}$ and the other is β.

By varying the Lagrangian (Equation (33)) with respect the deviation vector ψ^γ, we have the following:

$$\frac{\partial L}{\partial \psi^\gamma} = g_{\mu\nu}(U^\mu + \beta c^\mu)U^\alpha \nabla^\nu_{.\gamma\alpha}, \tag{36}$$

$$\frac{\partial L}{\partial \dot{\psi}^\gamma} = g_{\mu\gamma}(U^\mu + \beta c^\mu), \tag{37}$$

$$\frac{d}{d\tau}\frac{\partial L}{\partial \dot{\psi}^\gamma} = g_{\mu\gamma}\frac{dU^\mu}{d\tau} + g_{\mu\gamma,\sigma}U^\mu U^\sigma + \beta\frac{dc_\gamma}{d\tau}. \tag{38}$$

Substituting from Equations (36) and (38) into the Euler-Lagrange equation

$$\frac{d}{d\tau}\frac{\partial L}{\partial \dot{\psi}^\gamma} - \frac{\partial L}{\partial \psi^\gamma} = 0, \tag{39}$$

we get the following:

$$g_{\mu\gamma}\frac{dU^\mu}{d\tau} + g_{\mu\gamma,\sigma}U^\mu U^\sigma + \beta\frac{dc_\gamma}{d\tau} - g_{\mu\nu}(U^\mu + \beta c^\mu)U^\alpha \nabla^\nu_{.\gamma\alpha} = 0, \tag{40}$$

from metricity (Equation (20)), and we obtain the following:

$$\dot{U}^\mu + \nabla^\mu_{.\gamma\alpha}U^\alpha U^\gamma = -\beta g^{\mu\nu}\dot{c}_\nu + \beta c_\nu \nabla^\nu_{.\gamma\alpha}U^\alpha g^{\gamma\mu}. \tag{41}$$

Using Equations (18) and (27), Equation (41) can be written, explicitly, in the following form:

$$\frac{dU^\mu}{d\tau} + \begin{Bmatrix} \mu \\ \alpha\beta \end{Bmatrix} U^\alpha U^\beta = -b\gamma^\mu_{.(\alpha\beta)}U^\alpha U^\beta - \beta g^{\nu\mu} c_{\underset{+}{\nu\|\alpha}} U^\alpha. \tag{42}$$

It is obvious that if $b = 0$ and $\beta = 0$, the modified Equation (42) reduces to the geodesic Equation (3). Also, if $b = 0$, this equation tends to Equation (7) if we use the conventional method. When the electromagnetic sector is switched off (i.e., $\beta = 0$), the parameterized path (Equation (42)) reduces to Equation (30). We can take the parameter $\beta = e/m$, where e is the electron charge and m is the electron mass. This is done for dimensional consideration of the Lagrangian (Equation (33)). Note that c^μ is considered as a geometric representation of the electromagnetic potential.

We can rewrite Equation (42) as follows:

$$\frac{dU^\mu}{d\tau} + \begin{Bmatrix} \mu \\ \alpha\beta \end{Bmatrix} U^\alpha U^\beta = -b\gamma^\mu_{.(\alpha\beta)}U^\alpha U^\beta - \beta g^{\nu\mu}U^\alpha F_{\nu\alpha} - \beta g^{\nu\mu}U^\alpha c_{\underset{-}{\alpha\|\nu}}, \tag{43}$$

where

$$F_{\nu\alpha} \overset{\text{def.}}{=} c_{\nu,\alpha} - c_{\alpha,\nu}. \tag{44}$$

Important note: It is to be considered that Equation (44) is a field equation to be solved and a definition after the solution. This point will be more discussed in Section 5.

4. Linearization of the New Path Equation

In the context of geometrization philosophy, the BB of the geometry are considered as the field variables of the theory. So, in order to gain more physical meaning from the two-parameter geometric Equation (43), we are going first to linearize it. We consider a weak and a static unified field together with a slowly moving particle in this field; i.e., we assume the following:

(1) The BB ($\underset{i}{\lambda}_\mu$) of the PAP geometry in the following form:

$$\underset{i}{\lambda}_\mu = \delta_{i\mu} + \epsilon \underset{i}{h}_\mu, \tag{45}$$

where $\underset{i}{h}_\mu$ represents a set of functions of the coordinates causing deviation from Euclidean geometry. We expand all geometric quantities needed to the first order in ϵ and neglecting higher orders.

(2) A static unified field, which means that

$$\frac{\partial \underset{i}{\lambda}_{\mu}}{\partial x^{0}} = 0. \tag{46}$$

(3) A slowly moving particle, i.e., $U^0 \simeq 1$, $U^1 \sim U^2 \sim U^3 \sim \varepsilon$. Also, $O(\varepsilon^2)$ and $O(\varepsilon\epsilon)$ and higher order can be neglected

As a consequence of Equation (45), we get the following :

$$\underset{i}{\lambda}^{\mu} = \delta_{i\mu} - \epsilon \underset{i}{h}^{\mu}. \tag{47}$$

Substituting from Equations (45) and (47) into Equations (15) and (16), we obtain the following:

$$\begin{aligned} g_{\mu\nu} &= \delta_{\mu\nu} + \epsilon y_{\mu\nu}, \\ g^{\mu\nu} &= \delta_{\mu\nu} - \epsilon y_{\mu\nu}, \\ y_{\mu\nu} &\overset{\text{def.}}{=} \left(\underset{\mu}{h}_{\nu} + \underset{\nu}{h}_{\mu} \right). \end{aligned} \tag{48}$$

The linearized symmetric part of the contortion tensor $\gamma^{\mu}_{.\alpha\beta}$ will be in the following form:

$$\gamma^{\mu}_{.(\alpha\beta)} = \frac{\epsilon}{2} \left(y_{\alpha\beta,\mu} - \underset{\alpha}{h}_{\mu,\beta} - \underset{\beta}{h}_{\mu,\alpha} \right), \tag{49}$$

and also, the first order of the contracted torsion or contortion becomes the following:

$$c_{\mu} = \epsilon \left(\underset{\alpha}{h}_{\mu,\alpha} - \underset{\alpha}{h}_{\alpha,\mu} \right). \tag{50}$$

Recalling Equations (28) and (44), Equation (43) becomes the following:

$$\frac{dU^{\mu}}{d\tau} + \left\{ \begin{matrix} \mu \\ \alpha\beta \end{matrix} \right\} U^{\alpha} U^{\beta} = -b\gamma^{\mu}_{.(\alpha\beta)} U^{\alpha} U^{\beta} - \beta g^{\nu\mu} \left(c_{\nu,\alpha} - c_{\sigma} \left\{ \begin{matrix} \sigma \\ \alpha\nu \end{matrix} \right\} - bc_{\sigma}\gamma^{\sigma}_{\nu\alpha} \right) U^{\alpha}. \tag{51}$$

Applying the assumptions 1, 2, and 3 to the equation of motion (Equation (51)), we get the following:

$$\begin{aligned} \frac{dU^{\mu}}{dt} &= -\left\{ \begin{matrix} \mu \\ 00 \end{matrix} \right\} - b\gamma^{\mu}_{.00} \\ &= \frac{\epsilon}{2} y_{00,\mu} - b\frac{\epsilon}{2} y_{00,\mu}, \end{aligned} \tag{52}$$

which can be written in terms of Newtonian potential as follows:

$$\frac{d^2 x^a}{dt^2} = (1-b)\Phi_{,a}, \quad a = 1, 2, 3, \tag{53}$$

where

$$\Phi \overset{\text{def.}}{=} \frac{\epsilon}{2} y_{00}, \tag{54}$$

is the Newtonian gravitational potential.

5. Discussion and Concluding Remarks

In the framework of unified field theories, we expect to produce gravity from electromagnetism and vice versa. The first production is well known since Einstein-Maxwell theory [1]. Nature provides us with some evidences for producing electromagnetism from gravity, since most (if not all) celestial objects have magnetic fields of different orders while they are all electrically neutral. Some authors [8] have found a theoretical relation between the magnetic field and some gravitational properties. Other authors [9] have used this relation to interpret the huge magnetic field producing gamma ray bursts.

Now, in the present work, we are dealing with a pure geometric theory unifying gravity and electromagnetism [10]. So, we expect both types (as will appear in this discussion, two types of electromagnetism appear in the Universe.) of electromagnetism to be present in the theory that this will be clear in applications. In what follows, we call the first type as a cosmic electromagnetic field and the second type as a conventional electromagnetic field. Both will affect the motion of particles moving in the unified field. As a usual procedure, we solve the field equations before solving the equations of motion as usually done in GR. In the present work, the field equations [10] are in general sixteen in number while the equations of motion (Equation (43)) are four. After solving the field equation, sixteen field variables become known functions of the coordinate. Afterwards, we solve the equations of motion (Equation (43)) to get the components of the acceleration (or velocity) of the moving particles.

In the context of the geometrization philosophy, any field theory contains two types of equations. The first type controls the behaviour of the field (field equations) originated from the differential identities of the geometry used. The second type governs the motion of a particle in the field mentioned above (equations of motion). For example, in general relativity, the field equations emerged from Bianchi differential identity of Riemannian geometry, while the equations of motion are the general curves of the same geometry (geodesic).

In the present work, we use another theory [10] written in the PAP geometry [7]. Here, we derive the equation of the general curve in the PAP geometry (Equation (43)) which is used as an equation of motion of the theory. The field theory completed here has curvature and anticurvature. It has been shown that curvature gives rise to gravity and anticurvature gives rise to antigravity [11]. These theoretical predictions of Equation (30), in its linearized form, have been supported by interpreting the discrepancy of the COW experiment [12–14], which is verified in 2000.

Equation (43) is the general equation of motion for an electrically charged and spinning test particle moving in a general field unifying gravity and electromagnetism [10]. This equation contains two parameters b and β. The first parameter is related to the quantum spin of the moving particle and the second is related to the electric charge of the moving particle. Equation (43) has the following properties.

(1) If the two parameters b (Equation (32)) and β (Equation (6)) simultaneously vanish identically; i.e., the moving particle is a scalar (electrically neutral and has zero quantum spin). In this case, Equation (43) reduces to an ordinary geodesic (Equation (3)) of Riemannian geometry

(2) If the parameter β (or e) vanishes alone, in this case, Equation (43) reduces to Equation (30) for a spinning particle

(3) If the parameter $b = 0$, then Equation (43) will reduce to the following:

$$\frac{dU^{\mu}}{d\tau} + \left\{ \begin{matrix} \mu \\ \alpha\beta \end{matrix} \right\} U^{\alpha} U^{\beta} = -\beta g^{\nu\mu} U^{\alpha} F_{\nu\alpha} - \beta g^{\nu\mu} U^{\alpha} c_{\alpha;\nu}, \tag{55}$$

which is the equation of motion for a charged particle derived using the Bazanski scheme

In the present work, we applied general scheme called linearization of a nonlinear field theory. This scheme is applied after generalization to get the forces affecting the motion of an electrically charged and spinning test particle in a field unifying gravity and electromagnetism.

The conventional electromagnetic field can be presented together with a cosmic electromagnetic field in the context of some terrestrial experiments. This result gives the geometric interpretation of the Aharonov-Bohm effect. To get same result from the present work, more efforts are needed.

Data Availability

We have no data used in the manuscript.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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