International Astronomy and Astrophysics Research Journal

3(2): 22-32, 2021; Article no.IAARJ.69959

Law and Equation of Interplanetary Relationship and its Applications

Rajat Saxena[1*]

[1]*Independent Researcher, India.*

Author's contribution

The sole author designed, analyzed, interpreted and prepared the manuscript.

Article Information

Editor(s):
(1) Prof. Magdy Rabie Soliman Sanad, National Research Institute of Astronomy and Geophysics, Egypt.
(2) Prof. Hadia Hassan Selim, National Research Institute of Astronomy and Geophysics, Egypt.
Reviewers:
(1) Nelson Falcon, Universidad de Carabobo, Venezuela.
(2) J. J Rawal, Indian Planetary Society, India.
(3) M. G. Abrahamyan, Erevan State University, Armenia.
(4) Maziar Naderi, Tehran University of Medical Sciences, Iran.
Complete Peer review History: https://www.sdiarticle4.com/review-history/69959

Original Research Article

Received 01 May 2021
Accepted 06 July 2021
Published 13 July 2021

ABSTRACT

In classical theory, it is believed that physical characteristics of a planet like the axial tilt, the orbit radius, and the rotation period are independent quantities, meaning they are not directly related to each other. However, this paper will discuss a simple yet elegant equation that shows all these quantities are interlinked. The Interplanetary Relationship equation shows that for a planet, the ratio of axis tilt to the product of orbit radius and rotation period is a constant. The value of this constant is the topic of further discussion in the paper. The equation is a subject of beauty, as it relates quantities that seem to be independent of each other. The law and its equation have various applications, which will be discussed further in the paper and improve our understanding of the solar system.

Keywords: Interplanetary relationship; axis tilt; orbit radius; rotation period.

1. INTRODUCTION

In the last nine centuries, science has been formulating its surroundings, and we have been quite successful at it. We have advanced so much that now we have information regarding the planets

**Corresponding author: E-mail: sunny2003july@gmail.com;*

in our solar system. So we have been rewinding time to see how our solar system was formed.

Current research dictates that many planetesimals collided, combined, and grew to form our modern-day stable planets. Also, it is believed that these random collisions resulted in giving planets their physical characteristics.

In 1945, in one of the private letters, Albert Einstein said, "God does not play dice with the universe" [1], which means nothing happens randomly in the universe; there is always a reason. With the help of the equation (which we will discuss ahead) and its law, we hope to show that the planets do not take up their physical characteristics randomly. Also, we will see how quantities that seem to be unrelated and independent are directly linked to each other.

Repeated coincidences form patterns, and we humans understand the universe using these patterns. This also implies that there are no random coincidences in nature; there is always a reason, a higher governing body. The sciences aim to uncover this reason. The above philosophical statements best describe the paper's findings as the law was developed by observing frequently repeated coincidence.

The Interplanetary Relationship equation is not derived from any existing laws or formulas. It is the first of its kind. Hence, the introduction does not include references to previous studies.

For background on interplanetary relationship, check the paper mentioned in the references [2].

Before we elaborate on the law and the equation of Interplanetary Relationship, we will discuss another critical relation that will help us understand the latter.

2. INTRODUCTION TO LUNAR CONSTANT

For a planet, the Lunar constant is the ratio of the total number of moons to the orbit radius.

$$\boldsymbol{\eta} = \frac{\boldsymbol{n}}{\boldsymbol{r}} \tag{1}$$

Where,
$\eta = Lunar\ Constant$
$n = Total\ number\ of\ moons$
$r = Average\ distance\ between\ a\ planet\ and\ the\ Sun$

The total number of moons is a dimensionless quantity, and the unit of orbit radius is meters. Hence the unit of the lunar constant is m^{-1}.

Table 1. shows the calculation of the lunar constant for different planets

Name of Planet	Orbit Radius (*r*) $(10^9\ m)$	Number of Moons (*n*)	Lunar Constant $\left(\frac{n}{r}\right)$ (m^{-1})
Earth [3]	149.6	1	$\approx 10^{-11}$
Mars [4]	227.9	2	$\approx 10^{-11}$
Jupiter [5]	778.9	79	$\approx 10^{-10}$

If we observe the values in table 1, we will notice that the orbit radius and the total number of moons have some relation.

On rewriting equation one, we get the relation between the orbit radius and total number of moons as,

$$\boldsymbol{n} = \boldsymbol{\eta} \times \boldsymbol{r} \tag{2}$$

Equation two works more accurately for planets having many moons, as seen for the planet Jupiter.

Without the knowledge of Interplanetary Relationship law and equation, we cannot understand the relevance of the Lunar constant. The application of the lunar constant will get clear when we are acquainted with the interplanetary equation.

The table given below is for further reference.

Table 2. shows the values of the Lunar constant for different planets

Name of the Planet	The value of Lunar Constant
Mercury	$10^{-7}\ m^{-1}$
Venus	$10^{-10}\ m^{-1}$
Earth	$10^{-11}\ m^{-1}$
Mars	$10^{-11}\ m^{-1}$
Jupiter	$10^{-10}\ m^{-1}$
Neptune	$10^{-10}\ m^{-1}$

Note: We did not consider Uranus and Saturn to understand the lunar constant because these planets have many undiscovered moons. Hence, it would yield unexpected results from equation one.

Now that we have discussed the Lunar Constant, we can introduce ourselves to Interplanetary Relationship.

3. INTRODUCTION TO INTER-PLANETARY RELATIONSHIP

As the name suggests, Interplanetary Relationship is a mathematical relation between all planets. The law and its expression are derived from a parent equation.

The parent equation of interplanetary relationship is:

$$\boldsymbol{n}\left(\frac{\boldsymbol{\theta v}}{\boldsymbol{r^2 d}}\right) = \boldsymbol{\psi} \qquad (3)$$

Where,

For a planet
$n = Total\ number\ of\ moons$
$\theta = Axial\ tilt\ (rad)$
$v = Equatorial\ rotational\ velocity\ (m/s)$
$r = Orbit\ radius\ (m)$
$d = Diameter\ of\ a\ planet\ (m)$
$\psi = Solar\ interplanetary\ constant$

The unit of the solar interplanetary constant can be derived from the parent equation itself.
By substitution we get,

$$\psi = \left(\frac{rad\ \times ms^{-1}}{m^2 \times m}\right)$$
$$= rad(m^{-2}s^{-1})$$

Therefore the unit of the solar interplanetary constant is $rad(m^{-2}s^{-1})$.

The constant (ψ) is called the solar interplanetary constant, and its value is constant only in our solar system. Using the interplanetary relationship law and equation for other planetary systems is the topic for discussion ahead.

The parent equation might seem absurd initially. However, we will get aware of its validity when we substitute the physical characteristics of real planets to obtain the value of ψ.

4. FINDING THE VALUE OF Ψ VIA SUBSTITUTION

Note:

By trial and error, Kepler discovered three empirical laws of Planetary Motion [6]. Similarly, the parent equation was developed by trial and error. Hence, the only way of verifying it is by substitution of values in it.

1. For Earth [3],
$n = 1$
$\theta = 0.410\ rad$
$v = 465.1\ m/s$
$r = 149.6\ \times 10^9\ m$
$d = 12742\ \times 10^3\ m$

On substitution in equation three, we get,

$$\psi = 1\left(\frac{0.410 \times\ 465.1}{(149.6 \times 10^9)^2 \times 12742 \times 10^3}\right)$$

$$\psi = 6.68 \times 10^{-28}\ rad(m^{-2}s^{-1})$$

2. For Mars [4],
$n = 2$
$\theta = 0.436\ rad$
$v = 241.7\ m/s$
$r = 227.9\ \times 10^9\ m$
$d = 6779\ \times 10^3\ m$

On substitution in equation three, we get,

$$\psi = 1\left(\frac{0.436 \times\ 241.7}{(227.9 \times 10^9)^2 \times 6779 \times 10^3}\right)$$

$$\psi = 6.76 \times 10^{-28}\ rad(m^{-2}s^{-1})$$

3. For Mercury [7],

In equation 3, we can see that the numerator is multiplied by the total number of moons. However, Mercury and Venus have no moons. Therefore, to make the parent equation functional for these planets, we will substitute equation 2 in equation 3.

$$n = \eta \times r \qquad (2)$$

$$n\left(\frac{\theta v}{r^2 d}\right) = \psi \qquad (3)$$

We get,

$$\eta \times r\left(\frac{\theta v}{r^2 d}\right) = \psi$$
$$\Rightarrow \eta\left(\frac{\theta v}{rd}\right) = \psi \qquad (4)$$

Substituting values for Mercury in equation 4,

For Mercury [7],
$\eta = 10^{-7}\,m^{-1}$
$\theta = 0.0005934\,rad$
$v = 3.026\,m/s$
$r = 50.726\ \times 10^{9}\,m$
$d = 4880 \times 10^{3}\,m$

On substitution in equation four, we get,

$$\psi = \left(\frac{10^{-7}\ \times 0.0005934 \times\ 3.026}{50.726\ \times 10^{9} \times 4880 \times 10^{3}}\right)$$

$$\psi = 7.155 \times 10^{-28}\,rad(m^{-2}s^{-1})$$

In the above example, the lunar constant replaces the total number of moons (*n*) in the equation. So even though Mercury has zero moons still the value of its lunar constant is not zero. As Carl Sagan said, "extraordinary claims require extraordinary evidence". The examples shown above and the table below is our extraordinary evidence. This evidence supports the parent equation of interplanetary relationship.

Table 3. shows the values of ψ for different planets

Name of the planet	Value of ψ $rad(m^{-2}s^{-1})$
Mercury	7.155×10^{-28}
Earth	6.68×10^{-28}
Mars	6.76×10^{-28}
Jupiter	6.4×10^{-28}
Neptune	6.2×10^{-28}

As we can see in the above table, all the planets have similar values of ψ, validating the parent equation. Using Table 3, we can now get the average value of ψ.

Calculating the average of ψ,

$$\psi = \frac{10^{-28} \times (7.155 + 6.68 + 6.76 + 6.4 + 6.2)}{5}$$

$$\boldsymbol{\psi = 6.638 \times 10^{-28}\,rad(m^{-2}s^{-1})}$$

We can use the above value of ψ as the standard value of the constant.

5. DERIVATION OF THE INTER-PLANETARY RELATIONSHIP LAW AND EQUATION

Equation 3 or the parent equation is very complicated, and hence forming a law out of it would be difficult. Also, the parent equation can further be simplified by substitutions.
Simplifying Equation 3:

We know,

$$n = \eta \times r$$

Also,

$$velocity = \frac{distance}{time}$$

Distance covered during one complete rotation done by a planet (around its axis) = Circumference of the planet at equator = πd

Time = time period of rotation = t

$$\therefore v = \frac{\pi d}{t}$$

On substituting the value of *n* and *v* in the parent equation, we get,

$$\eta r\left(\frac{\theta \times \frac{\pi d}{t}}{r^2 d}\right) = \psi$$

$$\Rightarrow \eta\left(\frac{\theta \times \pi}{r \times t}\right) = \psi$$

Transferring all constants on the RHS,

$$\left(\frac{\boldsymbol{\theta}}{\boldsymbol{r} \times \boldsymbol{t}}\right) = \frac{\boldsymbol{\psi}}{\boldsymbol{\eta\pi}} \tag{5}$$

Equation 5 is the interplanetary relationship expression and using it, we can form the Law of Interplanetary Relationship. On the RHS of the equation, we have ψ, η and π; these values are constants. Hence the ratio of axis tilt to the product of orbit radius and rotation period is a constant for a planet.

The Law of the Interplanetary Relationship is,

"For a planet, the ratio of axis tilt to the product of orbit radius and period of rotation is a constant."

Using the law and its equation, we can revolutionize our understanding of the solar system.

A statement is called an equation when the values of two mathematical expressions are equal. The two mathematical expressions in equation five are the LHS and RHS.

$$\underbrace{\left(\frac{\boldsymbol{\theta}}{\boldsymbol{r}\times\boldsymbol{t}}\right)}_{\text{LHS}} = \overbrace{\frac{\boldsymbol{\psi}}{\boldsymbol{\eta\pi}}}^{\text{RHS}}$$

By substituting the planet's physical characteristics in the LHS and RHS, we will obtain their values. The obtained result should be LHS ≈ RHS for equation five to be true.

6. VERIFICATION OF THE INTER-PLANETARY RELATIONSHIP EXPRESSION BY SUBSTITUTION

For Jupiter [5],
$\theta = 0.05462\ rad$
$t = 35760\ s$
$r = 776.82\ \times 10^9\ m$
$\psi = 6.638\ \times 10^{-28}\ rad(m^{-2}s^{-1})$
$\pi = 3.14$
$\eta = 10^{-10}m^{-1}$

Substituting values in equation 5,

$$LHS = \frac{0.05462}{776.82\ \times 10^9 \times 35760}$$

$$RHS = \frac{6.638\ \times 10^{-28}}{3.14 \times 10^{-10}}$$

$\therefore LHS = 1.96\ \times 10^{-18}\ rad(m^{-1}s^{-1})$
$\therefore RHS = 2.11 \times 10^{-18}\ rad(m^{-1}s^{-1})$
$\therefore LHS \approx RHS$

For Earth [3],
$\theta = 0.410\ rad$
$r = 149.6\ \times 10^9\ m$
$t = 86400\ s$
$\eta = 10^{-11}\ m^{-1}$
$\psi = 6.638\ \times 10^{-28}\ rad(m^{-2}s^{-1})$
$\pi = 3.14$

Substitution in equation 5,

$$LHS = \frac{0.410}{149.6\ \times 10^9 \times 86400}$$

$$RHS = \frac{6.638\ \times 10^{-28}}{3.14 \times 10^{-11}}$$

$\therefore LHS = 3.17\ \times 10^{-17}\ rad(m^{-1}s^{-1})$
$\therefore RHS = 2.11 \times 10^{-17}\ rad(m^{-1}s^{-1})$

Even though the RHS and LHS are very close to each other, they are not precisely equal. Here, LHS is greater than RHS. If we increase the value of the denominator in the LHS, then its overall value will decrease. Using equation one, we will substitute the value of the lunar constant in equation 5.

Transferring lunar constant on the LHS,

$$\eta\left(\frac{\theta}{r \times t}\right) = \frac{\psi}{\pi}$$

Using equation one, we will substitute the value of the lunar constant in the above equation.

$$\frac{n}{r}\left(\frac{\theta}{r \times t}\right) = \frac{\psi}{\pi}$$

$$\Rightarrow \boldsymbol{n}\left(\frac{\boldsymbol{\theta}}{\boldsymbol{r}^2 \times \boldsymbol{t}}\right) = \frac{\boldsymbol{\psi}}{\boldsymbol{\pi}} \qquad (6)$$

Now, we have increased the value of the denominator by a factor of *r*.

Substituting the values for Earth in equation 6.

$$LHS = 1 \times \left(\frac{0.410}{(149.6\ \times 10^9)^2 \times 86400}\right)$$

$$RHS = \frac{6.638\ \times 10^{-28}}{3.14}$$

$\therefore LHS = 2.07\ \times 10^{-28}\ rad(m^{-2}s^{-1})$
$\therefore RHS = 2.11 \times 10^{-28}\ rad(m^{-2}s^{-1})$
$\therefore LHS\ \approx RHS$

In the above two examples, we have seen how flawlessly the equation works.

Table 4. Shows the results when values for planets are substituted in equation 5 or 6.

Name of Planet	Obtained Value (LHS)	Expected Value (RHS)
Mercury	2.308×10^{-21} $rad(m^{-1}s^{-1})$	2.11×10^{-21} $rad(m^{-1}s^{-1})$
Venus	2.541×10^{-18} $rad(m^{-1}s^{-1})$	2.11×10^{-18} $rad(m^{-1}s^{-1})$
Earth	2.07×10^{-28} $rad(m^{-2}s^{-1})$	2.11×10^{-28} $rad(m^{-2}s^{-1})$
Mars	2.12×10^{-18} $rad(m^{-1}s^{-1})$	2.11×10^{-18} $rad(m^{-1}s^{-1})$
Jupiter	1.96×10^{-17} $rad(m^{-1}s^{-1})$	2.11×10^{-17} $rad(m^{-1}s^{-1})$
Neptune	1.90×10^{-18} $rad(m^{-1}s^{-1})$	2.11×10^{-18} $rad(m^{-1}s^{-1})$

We can see in table 4 that the obtained values and expected values are very close to each other, verifying the functionality of the Interplanetary Relationship Equation.

Table 5. Shows the deviation of Planets from the expected value

Name of the Planet	Deviation from the expected value$\lvert(\boldsymbol{RHS}) - (\boldsymbol{LHS})\rvert$
Mercury	$0.194 \times 10^{-21}\ rad(m^{-1}s^{-1})$
Venus	$0.431 \times 10^{-18}\ rad(m^{-1}s^{-1})$
Earth	$0.04 \times 10^{-28}\ rad(m^{-2}s^{-1})$
Mars	$0.01 \times 10^{-18}\ rad(m^{-1}s^{-1})$
Jupiter	$0.15 \times 10^{-17}\ rad(m^{-1}s^{-1})$
Neptune	$0.21 \times 10^{-18}\ rad(m^{-1}s^{-1})$

In Table 5, we can observe that the value of deviation is negligible; hence we can conclude that the Interplanetary Relationship Law and equations are correct and can be considered as scientific fact.

7. UNIVERSAL EQUATION OF INTERPLANETARY RELATIONSHIP

In terms of the diameter of the sun, the solar interplanetary constant can be rewritten as:

$$\boldsymbol{\psi} = \boldsymbol{\kappa} \times \boldsymbol{D_s} \qquad (7)$$

Where,
$\psi = Solar\ Interplanetary\ Constant$
$\kappa =\ Universal\ Interplanetary\ Constant$
$D_S = Diameter\ of\ the\ Sun$

We know,
The diameter of the Sun $(D_S) = 1.3927 \times 10^9\ m$

Also,
The value of solar interplanetary constant (ψ) = $6.638\ \times 10^{-28}\ rad(m^{-2}s^{-1})$

Using equation 7,

$$\kappa = \frac{6.638\ \times 10^{-28}}{1.3927 \times 10^9} rad(m^{-3}s^{-1})$$

$$\therefore \boldsymbol{\kappa} = \mathbf{4.76}\ \times \mathbf{10^{-37}}\ \boldsymbol{rad(m^{-3}s^{-1})}$$

Using equation 7, we can obtain the solar interplanetary constant in terms of the Sun's diameter. The solar interplanetary constant contains the word 'solar' because its value depends on the solar diameter, as shown in equation 7.

Rewriting the interplanetary relationship equation in terms of equation 7,

$$\left(\frac{\theta}{r \times t}\right) = \frac{\kappa \times D_s}{\eta\pi}$$

In a new planetary system, the only quantity which changes in the above equation is the diameter of the central star; the rest all remain the same (assuming κ is a universal constant). Thus, the equation which can adapt to the changing diameter of the central star is:

$$\left(\frac{\boldsymbol{\theta}}{\boldsymbol{r} \times \boldsymbol{t}}\right) = \frac{\boldsymbol{\kappa} \times \boldsymbol{D}}{\boldsymbol{\eta\pi}} \qquad (8)$$

Where,
$\kappa = Universal\ Interplanetary\ Constant$
$D = diameter\ of\ the\ central\ star$

Due to a lack of data on exoplanets and alien planetary systems, we cannot verify equation eight.

Now that we have thoroughly discussed interplanetary relationship law and equation, we will move forward to its applications.

Note:

The value of the solar interplanetary constant is,
ψ = $6.638\ \times 10^{-28}\ rad(m^{-2}s^{-1})$

The value of the universal gravitational constant is,
$G = 6.674 \times 10^{-11}\ m^3kg^{-1}s^{-2}$

Both these values are very similar. Therefore, intuition dictates expressing ψ in terms of G. However, we need to understand that ψ and G are very different quantities. G is constant universally whereas, ψ is constant only in our solar system. Hence, we cannot express ψ in terms of G. Their values being similar is just a beautiful coincidence.

8. APPLICATIONS OF INTERPLANETARY RELATIONSHIP

1. Using Kepler's Third Law of Planetary Motion with the Interplanetary Relationship Equation

Kepler's Third Law [8] states that the squares of the sidereal periods of the planets are directly proportional to the cubes of their mean distances from the Sun. The sidereal period is the time required by a body to revolve around another body; in the case of planets, that is, the duration of a year. The equation of Kepler's Third Law is

$$\frac{T^2}{r^3} = \frac{4\pi^2}{GM} \tag{9}$$

Where,
$T = Sidereal\ Period\ (s)$
$M = Mass\ of\ the\ Sun\ (Kg)$
$G = Universal\ Gravitational\ Constant$
$r = Radius\ of\ orbit\ (m)$

If we compare equation five and equation nine, we will notice that they do not have much in common. However, we can build a relation using the sidereal period and rotation period.

We know that the sidereal period is the duration of a year, and the rotation period is the duration of a day. Hence, for a planet, the ratio of the sidereal period to the rotation period is the number of days in a year. Thus, we get the relation as:

$$\boldsymbol{\alpha = \frac{T}{t}} \tag{10}$$

Where,
$\alpha = number\ of\ days$

We get the value of *T* from equation nine as,

$$T = 2\pi r\sqrt{\frac{r}{GM}}$$

Substituting the value of *T* in equation 10,

$$\alpha = \frac{2\pi r\sqrt{\frac{r}{GM}}}{t}$$

$$\Rightarrow t = \frac{2\pi r}{\alpha}\sqrt{\frac{r}{GM}}$$

Substituting the value of *t* in equation 5,

$$\left(\frac{\theta}{r \times \frac{2\pi r}{\alpha}\sqrt{\frac{r}{GM}}}\right) = \frac{\psi}{\eta\pi}$$

$$\Rightarrow \left(\frac{\theta\alpha\sqrt{GM}}{2r^2\sqrt{r}}\right) = \frac{\psi}{\eta}$$

On transferring all constants to RHS, we get,

$$\boldsymbol{\left(\frac{\theta\alpha}{r^2\sqrt{r}}\right) = \frac{2\psi}{\eta\sqrt{GM}}} \tag{11}$$

The above equation is a combined form of interplanetary relationship equation and Kepler's Third Law.

For testing the equation, we will substitute values for Mars in equation eleven.

For Mars [4],
$\theta = 0.436\ rad$
$\alpha = 669.14$
$r = 227.9\ \times 10^9\ m$
$\psi = 6.638\ \times 10^{-28}\ rad(m^{-2}s^{-1})$
$\eta = 10^{-11}\ m^{-1}$
$G = 6.674 \times 10^{-11}\ m^3kg^{-1}s^{-2}$
$M = 1.989 \times 10^{30}\ kg$

On substitution in equation 11, we get,

$$LHS = \frac{0.436 \times 669.14}{(227.9\ \times 10^9)^2 \times \sqrt{227.9 \times 10^9}}$$

$$RHS = \frac{2 \times 6.638 \times 10^{-28}}{10^{-11} \times \sqrt{6.67 \times 1.98 \times 10^{19}}}$$

$\therefore LHS = 1.17 \times 10^{-26}\ rad\ m^{-\frac{5}{2}}$
$\therefore RHS = 1.155 \times 10^{-26}\ rad\ m^{-\frac{5}{2}}$
$\therefore LHS \approx RHS$

Hence, equation eleven is verified.

The aim of combining Kepler's third law with the interplanetary equation was to include more quantities in the relationship as this will help us find more unknowns in our solar system and beyond.

Note:

The exercise ahead is done for fun and out of curiosity. Physics is not done to obtain some practical results; it is done for the sense of amazement.

We will express equation 11 in terms of gravitational force (between a planet and the Sun) and centrifugal force on a planet (due to rotation around the Sun). For this, we will rewrite equation 10 in terms of distance and velocity.

We know,

$$T = \frac{distance\ travelled\ in\ one\ orbit}{orbital\ velocity}$$
$$\Rightarrow T = \frac{2\pi r}{v_o}$$

Where,
$r = orbit\ radius$
$\therefore 2\pi r = distance\ travelled\ in\ 1\ year$
$v_o = Orbital\ velocity$

(Distance travelled in one year = distance travelled in one Orbit)

Similarly,

$$t = \frac{circumference\ of\ a\ planet}{rotational\ velocity}$$
$$\Rightarrow t = \frac{\pi d}{v}$$

Where,
$d = diameter\ of\ a\ planet$
$\therefore \pi d = distance\ travelled\ in\ one\ day$
$v = Equatorial\ rotational\ velocity$

Substituting the value of *t* and *T* in equation 10,

$$\alpha = \frac{\frac{2\pi r}{v_o}}{\frac{\pi d}{v}}$$
$$\Rightarrow \boldsymbol{\alpha} = \frac{\mathbf{2vr}}{\boldsymbol{dv_o}} \qquad (12)$$

The gravitational force between a planet and the Sun, according to Newton's Law of Gravitation [9] is,

$$F_g = \frac{GMm}{r^2}$$
$$\Rightarrow GM = \frac{F_g \times r^2}{m}$$
$$\therefore \sqrt{GM} = r\sqrt{\frac{F_g}{m}}$$

Where,
$F_g = Gravitational\ force$
$G = Universal\ gravitational\ constant$
$M = Mass\ of\ the\ Sun$
$m = Mass\ of\ the\ planet$
$r = Distance\ between\ the\ bodies$

Substituting the value of GM and α in equation 11.

$$\frac{\theta}{r^2\sqrt{r}} \times \frac{2vr}{dv_o} = \frac{2\psi}{\eta} \times \frac{1}{r}\sqrt{\frac{m}{F_g}}$$

On cancelling out the common terms we get,

$$\frac{\theta}{\sqrt{r}} \times \frac{v}{dv_o} = \frac{\psi}{\eta} \times \sqrt{\frac{m}{F_g}}$$

We know,

$$v = \frac{\pi d}{t}$$

Substituting the value of *v* in the above equation,

$$\frac{\theta}{\sqrt{r}} \times \frac{1}{dv_o} \times \frac{\pi d}{t} = \frac{\psi}{\eta} \times \sqrt{\frac{m}{F_g}}$$

Cancelling the common terms,

$$\frac{\theta}{\sqrt{r}} \times \frac{1}{v_o} \times \frac{\pi}{t} = \frac{\psi}{\eta} \times \sqrt{\frac{m}{F_g}}$$
$$\Rightarrow \frac{\theta\pi}{v_o t\sqrt{r}} = \frac{\psi}{\eta}\sqrt{\frac{m}{F_g}}$$

Transferring all constants to the RHS and all the variables to the LHS,

$$\frac{\theta\sqrt{F_g}}{v_o t\sqrt{rm}} = \frac{\psi}{\eta\pi}$$

Squaring both sides,

$$\left(\frac{\theta}{t}\right)^2 \frac{F_g}{{v_o}^2 mr} = \left(\frac{\psi}{\eta\pi}\right)^2$$

Multiplying the numerator and denominator of LHS with *r*

$$\left(\frac{\theta}{t}\right)^2 \frac{F_g r}{{v_o}^2 mr^2} = \left(\frac{\psi}{\eta\pi}\right)^2$$
$$\Rightarrow \left(\frac{\theta}{r \times t}\right)^2 \frac{F_g r}{{v_o}^2 m} = \left(\frac{\psi}{\eta\pi}\right)^2$$

We know,

$$F_c = \frac{m{v_o}^2}{r}$$
$$\Rightarrow \frac{1}{F_c} = \frac{r}{{v_o}^2 m}$$

Where,

$F_c = Centrifugal\ force\ on\ a\ planet\ due\ to\ rotation\ around\ the\ sun$

$$\therefore \left(\frac{\theta}{r \times t}\right)^2 \frac{F_g}{F_c} = \left(\frac{\psi}{\eta\pi}\right)^2$$

Applying square root on both sides,

$$\left(\frac{\theta}{r \times t}\right) \sqrt{\frac{F_g}{F_c}} = \left(\frac{\psi}{\eta\pi}\right)$$

From equation five, we know,

$$\left(\frac{\theta}{r \times t}\right) = \left(\frac{\psi}{\eta\pi}\right)$$

Cancelling equation five from both sides,

$$\sqrt{\frac{F_g}{F_c}} = 1$$

$$\sqrt{F_g} = \sqrt{F_c}$$

$$\therefore |F_g| = |F_c|$$

Gravitational force and centrifugal force having equal magnitude is already a well-established fact.

This exercise's goal was to be more comfortable and flexible in using the interplanetary relationship equation. Also, the results of this exercise agree with already existing facts proving the working of the interplanetary relationship equation.

2. Finding lunar constant for planets with no moons.

Planets like Mercury and Venus have no moons. However, the value of lunar constants for these planets is not zero. Therefore, we cannot use equation 1 for such planets. Hence, we use the interplanetary relationship equation.

Making the lunar constant as the subject in equation 5, we get,

$$\eta = \frac{\psi}{\pi}\left(\frac{r \times t}{\theta}\right)$$

Using the above equation, we can find the value of the lunar constant for planets with no moons.

Trying the equation for Mercury [7]
$\theta = 0.0005934\ rad$
$r = 50.726 \times 10^9\ m$
$t = 506704.40\ s$
$\psi = 6.638\ \times 10^{-28}\ rad(m^{-2}s^{-1})$
$\pi = 3.14$

Substituting the values for finding the Lunar Constant,

$$\eta = \frac{6.638\ \times 10^{-19} \times 50.726 \times 506704.40}{3.14 \times 0.0005934}$$

$$\therefore \eta = 1.09 \times 10^{-7}\ m^{-1}$$

$$\eta \approx 10^{-7}\ m^{-1}$$

This is how we find the Lunar constant for Planets with no moons.

3. Finding the values of a planet's physical characteristics.

Equation five and its derivatives include many quantities, which can be found easily by substitution. For a planet, the quantities that can be determined include:

1. Axis tilt
2. Orbit radius
3. The time period of rotation
4. Radius and diameter
5. Rotational Velocity
6. Orbital velocity
7. Planet mass
8. Gravitational force
9. Centripetal force
10. Number of moons
11. Sidereal period
12. Number of days in a year
13. Density
14. Solar mass
15. Universal gravitational constant
16. Diameter of the Sun
17. Lunar constant

This shows the broad application of the interplanetary relationship equation and its derivatives.

4. Relation between two planets belonging to the same planetary system.

Let us consider two planets (which belong to the same planetary system), A and B. We are going to establish a relationship between Planets A and B using equation five.

For Planet A,
Let θ_A be the axis tilt for Planet A
Let r_A be the orbit radius for Planet A
Let t_A be the rotation period for Planet A
Let η_A be the lunar constant for Planet A

We get,

$$\frac{\theta_A}{r_A \times t_A} = \frac{\psi}{\eta_A \pi}$$

On transferring the lunar constant to the LHS, we get,

$$\Rightarrow \frac{\theta_A \times \eta_A}{r_A \times t_A} = \frac{\psi}{\pi} \qquad (13)$$

For Planet B,
Let θ_B be the axis tilt for Planet B
Let r_B be the orbit radius for Planet B
Let t_B be the rotation period for Planet B
Let η_B be the lunar constant for Planet B

We get,

$$\frac{\theta_B}{r_B \times t_B} = \frac{\psi}{\eta_B \pi}$$

On transferring the lunar constant to the LHS, we get,

$$\Rightarrow \frac{\theta_B \times \eta_B}{r_B \times t_B} = \frac{\psi}{\pi} \qquad (14)$$

In Equation 13 and 14, the RHS is the same (ψ/π). Hence we can say,

$$\boldsymbol{\frac{\theta_A \times \eta_A}{r_A \times t_A} = \frac{\theta_B \times \eta_B}{r_B \times t_B}} \qquad (15)$$

The equation is true for any two planets belonging to the same planetary system.

All the vital applications have been discussed. The Interplanetary relationship equations have many other applications; the only limit being the human imagination.

9. CONCLUSION

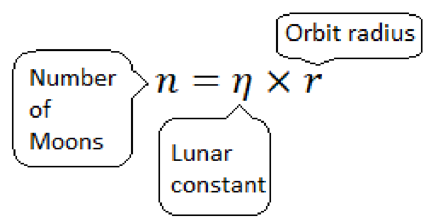
The Interplanetary Relationship equation is a new concept; hence, more research must be further dedicated. In science, answering one question often leads us to many more questions and this is how humanity evolves. The questions which remain unanswered are:

1. Validity of universal interplanetary relationship expression
2. Why the lunar constant is a non-zero quantity for planets having no moons?
3. Is the Interplanetary relationship law valid during the early days of the solar system?

More such questions will arise when more research is done on the interplanetary relationship.

10. SUMMARY

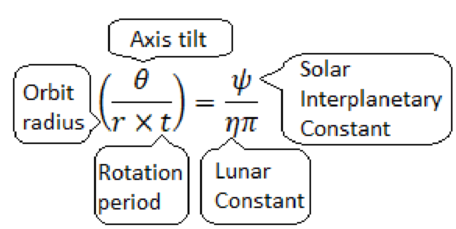
In the paper, we discussed many new relations. Initially, we discussed the lunar constant. For a planet, the Lunar Constant is the ratio of the total number of moons to the orbit radius. The equation can be rewritten as:



With the introduction of the lunar constant, we could begin our discussion on interplanetary relationship law and its expression.

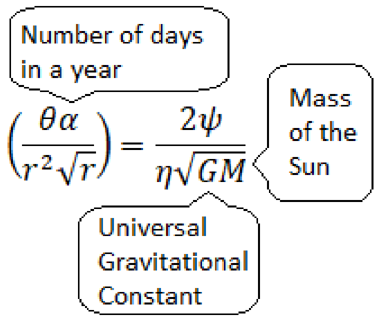
The law states that "For a planet, the ratio of axis tilt to the product of orbit radius and period of rotation is constant."

The expression is:



By substituting the physical characteristics of planets in the interplanetary equation, we verified its working and established it as a scientific fact. Now that we have familiarized ourselves with the equation, we will discuss its applications.

The substitution of Kepler's third law equation in the interplanetary relationship gave us:



Further, we discussed the quantities that can be derived using the interplanetary relationship equation.

ACKNOWLEDGEMENTS

This paper is dedicated to the one whom the equations describe, Krishna. The paper was possible only because of my family's unconditional love. I want to thank Edwina D'Souza, Siddharth Mathkar and Farhaan Ashfaq Ahmed Shaikh for reviewing this paper. Finally, I want to thank Sir Albert Einstein for leaving behind a great legacy.

COMPETING INTERESTS

The author has declared that no competing interests exist.

REFERENCES

1. Born I. The Born-Einstein Letters. Walker and Company, New York. 1971;1118.
2. Saxena R. Interplanetary relationship with some parameters of two planets based on the angular diameter of the sun. International Astronomy and Astrophysics Research Journal. 2020; 2(3):10-27. Available:https://www.journaliaarj.com/index.php/IAARJ/article/view/23
3. Williams D. Earth fact sheet; 2017. Available:https://nssdc.gsfc.nasa.gov/planetary/factsheet/earthfact.html Accessed 27 June 2021.
4. Williams D. Mars fact sheet; 2018. Available:https://web.archive.org/web/20200317184127/https:/nssdc.gsfc.nasa.gov/planetary/factsheet/marsfact.html Accessed 27 June 2021
5. Williams D. Jupiter fact sheet; 2021. Available:https://nssdc.gsfc.nasa.gov/planetary/factsheet/jupiterfact.html Accessed 27 June 2021
6. Young H. Freedman R. University Physics (14th Edition), Pearson Education, Noida Page. 1998; 4365
7. Williams D. Mercury Fact Sheet; 2020. Available:https://nssdc.gsfc.nasa.gov/planetary/factsheet/mercuryfact.html Accessed 27 June 2021.
8. Feynman R, Leighton R, Sands M. The Feynman lectures on physics. Dorling Kindersley, India. 1965;1:7.2.
9. Feynman R, Leighton R, Sands M. The Feynman lectures on physics. Dorling Kindersley, India. 1965;1:7.9.

Peer-review history:
The peer review history for this paper can be accessed here:
https://www.sdiarticle4.com/review-history/69959