British Journal of Education, Society & Behavioural Science
19(2): 1-12, 2017; Article no.BJESBS.30866
ISSN: 2278-0998

SCIENCEDOMAIN *international*
www.sciencedomain.org

Mathematics Teachers' Perceptions on Enhancing Students' Reasoning Skills in Mathematics

Mulugeta Atnafu Ayele[1*]

[1]Addis Ababa University, Addis Ababa, Ethiopia.

Author's contribution

The sole author designed, analyzed, interpreted and prepared the manuscript.

Article Information

DOI: 10.9734/BJESBS/2017/30866
Editor(s):
(1) Alina Georgeta Mag, Department of Private Law and Educational Science, University of Sibiu, Romania.
Reviewers:
(1) Julius S. Kasuni, Mount Kenya University, Kenya.
(2) Christine Krisnandari Ekowati, University of Nusa Cendana, Indonesia.
(3) Ekemini Akpan Titus, University of Uyo, Uyo, Nigeria.
Complete Peer review History: http://www.sciencedomain.org/review-history/17609

Original Research Article

Received 5th December 2016
Accepted 10th January 2017
Published 26th January 2017

ABSTRACT

The purpose of this study was to assess the perception of in-service mathematics teachers enhancing students' reasoning skills in mathematics. Exploratory survey design and quantitative research method were used. 102 in-service mathematics teachers were taken using stratified random sampling based on the mathematics teachers of postgraduate diploma in teaching (PGDT) and master programs. The data was collected by a four point rating scale, and analyzed by mean, standard deviation, correlation, independent sample t-test, one-way and two-way ANOVA. The major finding were, most of the mathematics teachers felt that they make reasoning as a focus in any mathematics class; Engage, develop and monitor students' mathematical understanding, skills, and reasoning; Constantly reflect on teaching practice, identify student progress and make instructional decisions; Apply inductive and deductive reasoning techniques and multiple representations to enhance the reasoning skills and support students to judge the validity of mathematical arguments and draw appropriate conclusions. In addition, the PGDT mathematics teachers had more experience in enhancing students' reasoning skills in mathematics than Master mathematics teachers (t=3.239, df=99, P=.002) and the preparatory mathematics teachers' response significantly negatively deviated from primary and secondary mathematics teachers in the variable enhancing students' reasoning skills in mathematics (F(2,98)=5.387, P=.006). But the short,

**Corresponding author: E-mail: ayatmu@yahoo.com;*

average and long service year mathematics teachers had nearly similar responses on enhancing students' reasoning skills in mathematics (F(2,98)=1.559, P=.215). As a conclusion, teachers select open-ended tasks that engage and develop students' mathematical thinking, understanding, skills, and reasoning; emphasizes the mathematical discourse in the classroom; use different assessments techniques to promote students reasoning and apply a variety of instructional techniques.

Keywords: Critical thinking; perception; program; level of teaching; service year; mathematics.

1. INTRODUCTION

Most research on mathematical education emphasizes the importance of mathematical reasoning as an integral part of doing mathematics [1]. According to the National Council of Teachers of Mathematics (NCTM) standards, mathematical reasoning requires the attainment of abilities to construct mathematical conjectures, develop and evaluate mathematical arguments, and select and use various types of representations. In mathematics, reasoning is used to solve problems, and also to decide whether an assertion is correct [1]. Students tend to engage in mathematical reasoning when they recognize that a logical inference (or series of inferences) is called for, recognize the type and degree of justification needed, and harness language(s), including mathematical terms and symbols, to create an explanation [1].

In Ethiopian education system reasoning is a focus in the curriculum of teaching mathematics. According to Ministry of Education (MoE) [2], the Ethiopian mathematics curriculum of grade 1 to 4 indicated that reasoning and creative activity provides a method of solving problems in everyday life situations and it is taught from a base of concrete experiences so that students learn to solve problems themselves. This curriculum also stated the purpose of the mathematics curriculum as to ensure that students develop an appreciation of how mathematics is used, an enjoyment of the reasoning and problem solving ability that its study develops and competence in its fundamental tools [2]. According to MoE [3], the Ethiopian mathematics curriculum of Grade 5 to 8 also indicated that the development of mathematical thinking does not linearly follow the acquisition of basic skills. This implies that instruction should provide opportunities for thinking and reasoning throughout the mathematics curriculum. The curriculum indicated that thinking and knowledge are not separate components of mathematical competency but rather are highly related. The quality of mathematical thinking and reasoning is dependent on what one knows, and the development of what one knows is dependent on mathematical thinking and hence students must be provided with opportunities to construct mathematical knowledge through thinking and reasoning and to think and reason with their mathematical knowledge, as they attempt to make sense of their experiences through real world problem solving [3].

Reasoning is defined as the process of thinking as an attempt an explanation in an attempt to show the relationship between two or more based on the properties, or certain laws that have been proven true through certain steps and ends with a conclusion [4]; The process of thinking according to the groove framework of certain thinking, thinking processes with opposite senses of observation or empirical observation, which produces a number of terms and propositions [5]. The term reasoning is a translation of reasoning which is defined as the process of reaching a logical conclusion based on facts and the relevant sources [6]; How to transform the information given in a specific in order to reach conclusions [7].

According to Keraf [8], reasoning as a thinking process which effort to correlate the evidences which understood that leads to the conclusion. The reasoning needs logical foundation. The reasoning in logic is not based on memorization but it is based on searching the reasoning that is previously constructed by students' mind. In the process of searching these reasoning, students have a discussing, sharing, interaction with other sides to draw a conclusion on a problem. One of the strategies that could stimulate students reasoning is problem solving strategy which makes the students compose an alternative solution. Applying this strategy the students have much opportunity to create new ideas to solve the problem, elucidate anything as detail as possible, observe the problem from any point of views, etcetera.

Shield and Galbraith [9] stated that much of the research literature has paid particular attention to the use of student writing activities as part of mathematical learning. They conducted a study in which they looked at the effects of writing in the math classroom, and if it enhanced students' learning. Specifically they focused on a scheme for coding student responses for analyzing student responses. More and more mathematics courses are requiring students to not only understand how to follow a particular procedure to solve a math problem, but also to understand why that particular procedure works. Much of the research indicates that for students to make connections and understand the mathematics deeply they must understand the conceptual side of the mathematics also. One of the ways that students can move to a more reflective stage of understanding is through the process of writing down their reasoning and mathematical thinking. Albert [10] indicated that the use of questioning used by both the teacher and students can help students become more proficient writers and develop a clear understanding of the math concepts.

One of the indicators of mathematical reasoning that can be solved through counter example strategy, where the students be asked to comprehend not only true example but also incorrect one. As Klymchuk [11] stated that counter-example is an example which shows that statement which being given (assumption, hypothesis proposition, and formulation) is incorrect.

Mevarech and Kramarski [12] suggested that developing mathematical reasoning in small-group activities must be structured to maximize the opportunities for each student to be engaged in questioning, elaboration, explanation, and other verbal communication through which students can express their ideas and group members can give and receive feedback. Albert [10] showed that when students worked collaboratively in a group they gained more of an insight into their own mathematical thinking, which helped them, develop their written responses.

Mevarech & Kramarski [12] and Lester, Garofalo, & Kroll [13] examined the effects of metacognitive training on mathematics reasoning. In these studies, the metacognitive training was based on Polya's [14] approach for solving mathematical problems. A major common element of these programs is training students who work in small groups to formulate and answer a series of self-addressed metacognitive questions that focus on the nature of the problem or task; The construction of relationships between previous and new knowledge and the use of strategies appropriate for solving the problem or task.

Calculators and computers are responsible for a rebirth of experimental mathematics [15]. They provide educators with wonderful tools for generating and validating patterns that can help students learn to reason mathematically. Computer games can help children master basic skills; intelligent tutors can help older students' master algebraic procedures. Many educators have argued that since programming enforces logical rigor, computer languages such as Logo and ISETL can help students learn to reason. Calculators and computers can either enhance mathematical reasoning or substitute for it, either develop mathematical reasoning or limit it.

NCTM [16] stated that at the 5-10 grade students, the math curriculum should include a lot of diverse experience that can reinforce and extend logical reasoning skills so that students can know damn apply deductive and inductive reasoning; understand and apply reasoning processes with special attention to the reasoning with proportions and graphs; make and evaluate conjectures-kunjektur and arguments logically; and assess the absorptive capacity and power of reasoning as part of mathematics.

1.1 Statement of the Problem

Developing the ability to reason mathematically is critical to students' success in the mathematics classroom. But regrettably, the development of reasoning among students has not been in the forefront of mathematics instruction across kindergarten to grade 12 (K-12). Therefore, it should not be surprising that mathematics students of all ages have difficulty in reflecting on the deeper meanings of what they are studying [17]. Scholars have already suggested that communication is a key element in the learning of mathematics [18]. Given the opportunity to discuss their thinking with peers and develop mathematical meaning through talk, all students stand a greater chance to develop reasoning competencies. Mathematics teacher may ignore the process of reasoning which is a time-consuming and not necessarily a linear process, in favour of building students' mathematical skills through teaching procedures.

1.2 Research Questions

The research questions for the study were:

1) To what extent the in-service mathematics teachers enhancing students' reasoning skills in mathematics?
2) Is there a significant difference in the responses of the in-service mathematics teachers enhancing students' reasoning skills in mathematics with respect to program?
3) Is there a significant difference in the responses of the in-service mathematics teachers enhancing students' reasoning skills in mathematics with respect to level of teaching?
4) Is there a significant difference in the responses of the in-service mathematics teachers enhancing students' reasoning skills in mathematics with respect to service year?
5) Are there significant interaction effect between program, level of teaching and service year on enhancing students' reasoning skills in mathematics?

2. MATERIALS AND METHODS

2.1 Research Design

The present study used exploratory survey design. The method used for this study was quantitative research method and it focused on a four point rating scale questionnaire.

2.2 Population and Sampling Method

The population for this study consisted of all 306 in-service mathematics teachers in Addis Ababa University. The 306 in-service mathematics teachers were divided into different strata such as: qualification (189 masters and 117 PGDT teachers), level of teaching (90 primary, 96 secondary, and 120 preparatory teachers), and teaching service year (114 short, 96 average, and 96 long service year teachers). 102 in-service mathematics teachers were selected from 306 in-service mathematics teachers using stratified random sampling based on qualification, level of teaching, and teaching service year. Using simple random sampling, 63 master teachers were selected from 189 master teachers, 39 PGDT teachers were selected from 117 PGDT teachers; 30 primary teachers were selected from 90 primary teachers, 32 secondary teachers were selected from 96 secondary teachers and 40 preparatory teachers were selected from 120 preparatory teachers and 38 short service year teachers were selected from 114 short service year teachers, 32 average service year teachers were selected from 96 average service year teachers, and 32 long service year teachers were selected from 96 long service year teachers.

2.3 Instruments of Data Collection

A four point rating scale on "In-service mathematics teachers' perceptions on enhancing students' reasoning skills in mathematics" which had 10 items, and the respondents were asked to respond to each item using a four point rating scale ranging strongly agree to strongly disagree such as strongly agree, agree, disagree and strongly disagree.

2.4 Validity and Reliability of the Instruments

The scale of teachers' perceptions on enhancing students' reasoning skills in mathematics was reviewed based on the comments of professionals for the face and content validity. A pilot study was conducted to determine the validity and reliability of the scale. Thirty in-service mathematics teachers which are not included in the main study were taken from Addis Ababa University. From the pilot study the alpha coefficient of Cronbach yielded 0.833 for the scale ‘enhancing students' reasoning skills in mathematics’. The Cronbach Alpha Coefficients of reliability for this variable indicated that they have high internal-consistency reliability.

2.5 Method of Analysis

Since the problem solving scale was an ordinal of four point rating scale and the skewness of the distribution for all 10 items lied between -1 and +1, this indicates that the data is not significantly different from normal. These justify that the variable is distributed approximately normally and we can use inferential statistics. Therefore, the data analysis techniques used for this study were Mean, Standard Deviation, Independent t-test, One way and Two way ANOVAs.

3. RESULTS AND DISCUSSION

3.1 Results

The first research question was to what extent the in-service mathematics teachers enhancing students' reasoning skills in mathematics? In

order to answer this question, 10 items were administered to the respondents to assess the items using a four point rating scale starting from strongly agree to strongly disagree. Table 1 presents the descriptive statistics of the responses of in-service mathematics teachers enhancing students' reasoning skills in Mathematics.

As can be seen from the above table of the items of reasoning skill of *'enhancing students' reasoning skills in mathematics'* are about 80% of the teachers making reasoning as a focus in mathematics class (mean=4.28), 79% selecting worthwhile tasks that develop students' mathematical reasoning (mean=4.18), 71% creating a classroom environment in which serious engagement in mathematical thinking is the norm (mean=3.96), 71% effectively orchestrating purposeful discourse aimed at encouraging students to reason and make sense of what they are doing (mean=3.96), 80% using a range of assessments to promote reasoning (mean=4.10), 74% constantly reflecting on teaching practice to be sure that the focus of the class is on reasoning (mean=3.99), 80% applying inductive and deductive reasoning techniques to build convincing mathematical arguments (mean=4.18), 64% developing conjectures on the basis of past experiences and intuition and test these conjectures using logic and/or probabilistic and statistical reasoning (mean=3.86), 85% helping students to explore the meaning and role of mathematical concepts, support them graphically or numerically, and verify them algebraically or geometrically (mean=4.34), and 76% supporting students to judge the validity of mathematical arguments and draw appropriate conclusions (mean=4.34) rated as the mean score were above average. For the aggregate of all the items of '*enhancing students' reasoning skills in mathematics*' the mean of the responses is 3.927, which is also above average.

Below is the analysis of the second research question that was 'Is there a significant difference of the in-service mathematics teachers enhancing students' reasoning skills in mathematics with respect to program?'

3.1.1 Program

In order to examine the significant differences of in-service mathematics teachers enhancing students' reasoning skills in mathematics with respect to their program independent sample t-test was used. Table 2 shows descriptive statistics and independent sample t-test for in-service mathematics teachers enhancing students' reasoning skills in mathematics with respect to their program.

From table 2, the descriptive statistics showed that the mean response of in-service PGDT mathematics teachers (4.283) had greater responses than that of in-service Masters mathematics teachers (3.948) for enhancing students' reasoning skills in mathematics. From the same table of an independent sample t-test indicated, t-value was statistically significant difference between in-service PGDT and Masters mathematics teachers in the cases of enhancing students' reasoning skills in mathematics ($t=3.239$, $df=99$, $P=.002$). Thus, in-service PGDT mathematics teachers had more experience in enhancing students' reasoning skills in mathematics than in-service Master mathematics teachers.

Below is the analysis of the third research question that was 'Is there a significant difference of the in-service mathematics teachers enhancing students' reasoning skills in mathematics with respect to level of teaching?'

3.1.2 Level of teaching

One-way ANOVA test was used to see the significance differences of the in-service mathematics teachers enhancing students' reasoning skills in mathematics with respect to level of teaching. Table 3 shows descriptive statistics and ANOVA test for in-service mathematics teachers enhancing students' reasoning skills in mathematics with respect to level of teaching.

Table 3 of the descriptive statistics showed that the responses of secondary in-service mathematics teachers had the highest and preparatory in-service mathematics teachers had the least mean responses in enhancing students' reasoning skills in mathematics. From the table, as ANOVA test indicated, F-value was statistically significant difference between the level of teaching groups for enhancing students' reasoning skills in mathematics ($F(2, 98) = 5.387$, $P = .006$). This indicates that primary, secondary and preparatory in-service mathematics teachers had significantly different in their responses of enhancing students' reasoning skills in mathematics.

Table 1. Descriptive statistics of the responses of in-service mathematics teachers enhancing students' reasoning skills in mathematics

No	Variables	Strongly disagree		Disagree		Agree		Strongly agree		Mean	SD
		f	%	f	%	f	%	f	%		
1	I can make reasoning as a focus in any mathematics class.	0	0	4	3.9	50	49.0	32	31.4	4.28	.697
2	I select worthwhile tasks that develop students' mathematical reasoning	0	0	4	3.9	58	56.9	23	22.5	4.18	.658
3	I create a classroom environment in which serious engagement in mathematical thinking is the norm	0	0	8	7.8	60	58.8	13	12.7	3.96	.749
4	I effectively orchestrating purposeful discourse aimed at encouraging students to reason and make sense of what they are doing	1	1.0	7	6.9	59	57.8	14	13.7	3.96	.798
5	I use a range of assessments to promote reasoning	0	0	6	5.9	61	59.8	21	20.6	4.10	.712
6	I constantly reflect on teaching practice to be sure that the focus of the class is on reasoning.	1	1.0	6	5.9	62	60.8	14	13.7	3.99	.757
7	I apply inductive and deductive reasoning techniques to build convincing mathematical arguments	0	0	5	4.9	56	54.9	26	25.5	4.18	.708
8	I develop conjectures on the basis of past experiences and intuition and test these conjectures using logic and/or probabilistic and statistical reasoning	2	2.0	11	10.8	48	47.1	17	16.7	3.86	1.003
9	I help students to explore the meaning and role of mathematical concepts, support them graphically or numerically, and verify them algebraically or geometrically	0	0	5	4.9	46	45.1	41	40.2	4.34	.745
10	I support students to judge the validity of mathematical arguments and draw appropriate conclusions	0	0	2	2.0	47	46.1	31	30.4	4.34	.615
	Enhancing students' reasoning skills in mathematics									**4.078**	**.529**

Table 2. Descriptive statistics and t-test for the responses of in-service mathematics teachers enhancing students' reasoning skills in mathematics with respect to their program

Components	Program	N	M	SD	t	df	P
Enhancing students' reasoning skills in mathematics	PGDT	39	4.283	.39230	3.239	99	.002
	Master	62	3.948	.56489			

Since the variable enhancing students' reasoning skills in mathematics made statistically significant differences with respect to level of teaching, Tukey HSD test is used in order to compare the mean difference of enhancing students' reasoning skills in mathematics with respect to the level of teaching primary, secondary and preparatory mathematics trainee teachers. Table 4 below indicates the Tukey HSD tests of the significant of mean difference of scores of enhancing students' reasoning skills in mathematics made statistically significant differences with respect to level of teaching.

The Tukey HSD Test from Table 4 above indicates that the preparatory in-service mathematics teachers significantly different from primary in-service mathematics teachers (MD = .35248, P = .014) and secondary in-service mathematics teachers (MD = .32781, P = .022) for the variable enhancing students' reasoning skills in mathematics. This indicates that preparatory in-service mathematics teachers' response significantly negatively deviated from primary and secondary in-service mathematics teachers in the variable enhancing students' reasoning skills in mathematics.

Below is the analysis of the fourth research question that was 'Is there a significant difference of the in-service mathematics teachers enhancing students' reasoning skills in mathematics with respect to service year?'

3.1.3 Service year

One-way ANOVA test was used to see the significance differences in in-service mathematics teachers enhancing students' reasoning skills in mathematics with respect to service year. Table 5 shows descriptive statistics and ANOVA test for in-service mathematics teachers enhancing students' reasoning skills in mathematics with respect to service year.

Table 5 of the descriptive statistics showed that the mean responses of short teaching service year of the in-service mathematics teachers had the highest mean whereas the average teaching service year of the in-service mathematics teachers had the least mean on enhancing students' reasoning skills in mathematics. From Table 5, as ANOVA test indicated, F-value was not statistically significant difference between the service year groups for enhancing students' reasoning skills in mathematics (F(2, 98) = 1.559, P = .215). This indicated that short, average and long teaching service year of the in-service mathematics teachers had nearly similar mean responses on the variable enhancing students' reasoning skills in mathematics.

Table 3. Descriptive statistics and ANOVA test for the responses of in-service mathematics teachers enhancing students' reasoning skills in mathematics with respect to level of teaching

Components	Level of teaching	N	M	SD	F	P
Enhancing students' reasoning skills in mathematics	Primary	30	4.2214	.40832	5.387	.006
	Secondary	32	4.1968	.44720		
	Preparatory	39	3.8689	.61105		

Table 4. Tukey test of the significant mean difference of scores of enhancing students' reasoning skills in mathematics with respect to level of teaching

Components	Region (I)	Region (J)	MD (I-J)	SE	P
Enhancing students' reasoning skills in mathematics	Primary	Secondary	.02468	.12893	.98
		Preparatory	.35248(*)	.12320	.014
	Secondary	Preparatory	.32781(*)	.12101	.022

** The mean difference is significant at the .05 level.*

Table 5. Descriptive statistics and ANOVA test for the responses of in-service mathematics teachers enhancing students' reasoning skills in mathematics with respect to service year

Components	Service year	N	M	SD	F	P
Enhancing students' reasoning skills in mathematics	Short	38	4.1660	.47382	1.559	.215
	Average	31	4.1038	.52209		
	Long	32	3.9470	.58609		

Table 6. Analysis of Variance for in-service mathematics teachers enhancing students' reasoning skills in mathematics as a function of program, level of teaching and service year

Variable and source	df	Mean square	F	P	Partial Eta Squared
Enhancing students' reasoning skills in mathematics					
Program	1	.172	.677	.413	.008
Level of teaching	2	.123	.483	.618	.011
Service year	2	.078	.308	.736	.007
Program * Level of teaching	1	.626	2.464	.120	.027
Program * Service year	1	.154	.342	.215	.016
Level of teaching * Service year	3	.127	.498	.684	.017
Program * Level of teaching * Service year	1	.171	.463	.099	.012
Error	89	.254			

Below is the analysis of the fifth research question that was 'Are there significant interaction effects between program, level of teaching and service year on enhancing students' reasoning skills in mathematics?'

The GLM Univariate procedure provides an analysis for main and interaction effects with the dependent variable of enhancing students' reasoning skills in mathematics.

The findings of the GLM Univariate (Table 6) yielded there was no main effect of program on enhancing students' reasoning skills in mathematics (F=.677, P=.413, eta^2=.008); there was no main effect of level of teaching on enhancing students' reasoning skills in mathematics (F=.483, P = .618, eta^2 = .011); and also there was no main effect of service year on enhancing students' reasoning skills in mathematics (F=.308, P=.736, eta^2=.007). Furthermore, there were not significant interaction effect between program, level of teaching, and service year (F=.463, P=.099, eta^2=.012); between program and level of teaching (F=2.464, P=.120, eta^2=.027); between program and service year (F=.256, P=.24, eta^2=.003); and between level of teaching and service year (F=.342, P=.215, eta^2=.017) on enhancing students' reasoning skills in mathematics. Thus it can be concluded that program, level of teaching, and service year were not significantly related with the dependent variable enhancing students' reasoning skills in mathematics, and in all cases according to Cohen (1988), the eta values indicate that the effect is very small.

3.2 Discussion

The key to improved mathematical reasoning is nurturing a student's ability to make thoughtful and defensible judgments. By tweaking our goals and questions, teachers can help students develop powerful tools to deepen their mathematical reasoning. Teachers can enhance student reasoning in mathematics by: Including an evaluative (student decision-making) component in the task or question; Being very specific in the questions asked; Articulating the criteria students should consider and requiring precisely explained evidence, reasoning or proof to support a conclusion. In this study most of the in-service mathematics teachers responded that they develop conjectures on the basis of past experiences and test these conjectures; Select tasks and create a classroom environment that help to engage and develop students' mathematical reasoning; Effectively orchestrate purposeful discourse aimed at encouraging students to reason; Use a range of assessments to promote reasoning; Constantly reflect on teaching practice to be sure that the focus of the mathematics class is on reasoning; Apply inductive and deductive reasoning techniques; help students to explore the mathematical concepts, support them graphically or numerically, and verify them algebraically or geometrically; and support students to judge the validity of mathematical arguments and draw appropriate conclusions. The aggregate average value of all the items of enhancing students' reasoning skills in mathematics is above average. Comparing the responses of teachers with respect to program of teaching, level of teaching and teaching service year: the in-service PGDT mathematics teachers had more experience in enhancing students' reasoning skills in mathematics than in-service Master mathematics teachers and the preparatory in-service mathematics teachers' response significantly negatively deviated from primary and secondary in-service mathematics teachers in the variable enhancing students' reasoning skills in mathematics. But the short, average and long

service year in-service mathematics teachers had nearly similar responses on the variable enhancing students' reasoning skills in mathematics. Moreover, program, level of teaching, and service year were not significantly related with the variable enhancing students' reasoning skills in mathematics.

Teachers develop conjectures on the basis of past experiences and test these conjectures; Select tasks that help to engage and develop students' understanding, skills, and reasoning; and support students to judge the validity of mathematical arguments and draw appropriate conclusions. Teacher moves specifically focuses on verbal moves, peripheral interventions such as task design/initiation of tasks and listening play a critical role in establishing a mathematical community and promoting student autonomy. Task selection or design should be incorporated in the teachers' planning of classroom teaching. Many researchers have emphasized the importance of task features in promoting reasoning and understanding [19,20]. Challenging, open-ended tasks are open to multiple representations and multiple strategies for solutions [20]. Goos [21] described as classrooms where students learn to talk and work mathematically by participating in mathematical discussions, proposing and defending arguments, and responding to the ideas and conjectures of their peers.

Teachers create a classroom environment in which serious engagement in mathematical thinking is the norm and effectively orchestrate purposeful discourse aimed at encouraging students to reason. Teacher interventions are a critical component of creating an environment that promotes the sharing of ideas in a learning community. Thoughtful interventions, implemented according to students' developing ideas, allow students to take ownership of their learning and solutions. According to Maher and Martino [22], by minimizing the teacher's role during initial exploration, students are more likely to engage in mathematical discourse, share representations, co-construct ideas and justifications, and ultimately take a more active role in their own learning. In this way, teachers can facilitate more elegant, clear explanations which lead to detailed, efficient representations and ultimately to further refined arguments by students [23]. Mathematical discourse can be promoted as students work together on tasks. After posing tasks, the teacher encourages students to begin to build their justifications and share ideas. During this phase the teacher engages in observation and careful listening in order to estimate how children are thinking about their solutions. Based on the type of task posed, the teacher initiates specific moves to promote reasoning and understanding.

Teachers use a range of assessments to monitor and promote reasoning, both in identifying student progress and in making instructional decisions and constantly reflect on teaching practice to be sure that the focus of the mathematics class is on reasoning. Mollborn & Hoekstra [24] suggested that one approach of improving student outcomes has been generating critical and reflective thinking skills among students; Mansoor, and Pezeshki [25] showed that in applying critical thinking in school settings, it is necessary to develop thinking skills, construct and evaluate arguments, detect common mistakes in reasoning and solving problems systematically; and Facione [26] identified six cognitive skills as central to the concept of critical thinking, among these were reflection and evaluation. Teachers questioning plays a crucial role in promoting student understanding, construction of new knowledge, as well as the sharing of ideas [27]. Martino and Maher [28] stress the importance of teacher questioning in creating an environment that promotes mathematical understanding and problem solving. Sahin and Kum [29] suggest that teachers should be cognizant of the types of questions they are asking and their purpose of asking these questions. The teacher may ask the learner to interpret a given answer or clarify on the answer given hence mathematical reasoning maybe developed depending on the type of questions asked by the teacher. The teacher's questions during the lesson may or may not give learners an opportunity to communicate their mathematical reasoning. If the teacher asks questions which require the learner to give a short and direct response, without asking the 'why' question as a follow up, then learners will not communicate their reasoning. The learners have to be able to explain their solutions, then teachers have to ask the learners for their explanations. Skilful questioning of student thinking and monitoring of student problem solving can provide teachers with a deeper understanding of the development of student's mathematical ideas and help advance student mathematical growth.

Teachers apply inductive and deductive reasoning techniques; Help students to explore

the mathematical concepts, support them graphically or numerically, and verify them algebraically or geometrically. In order to improve the reasoning skill of students, teachers should apply appropriate varieties of teaching methods, activities and aids such as: Formulate counter example and non-routine problems [11]; Students work in collaborative learning [30]; Metacognitive training on mathematics reasoning [31]; Use of calculators and computers in the teaching of mathematics [15]; Apply inductive and deductive reasoning techniques [16].

4. CONCLUSIONS AND RECOMMENDA-TIONS

The finding of this study indicate that most of the mathematics teachers make reasoning as a focus in their teaching by selecting worthwhile tasks, apply inductive-deductive reasoning, graphically or numerically techniques, classroom discourse, engaging students in mathematical thinking, and use a variety of assessments techniques. But there is a significant number of teachers have difficulty in enhancing students' reasoning skills in mathematics. In addition, the PGDT mathematics teachers had more experienced than the master mathematics teachers, and the preparatory mathematics teachers had less experienced than the primary and secondary mathematics teachers in enhancing students' reasoning skills in mathematics; but service year did not affect in enhancing students' reasoning skills in mathematics. Moreover, program, level of teaching, and service year were not significantly related with the variable enhancing students' reasoning skills in mathematics. In order to enhance the reasoning skills of the students, teachers should make reasoning as a focus in mathematics classroom by selecting open-ended tasks that engage and develop students' mathematical thinking, understanding, skills, and reasoning; emphasizes classroom discourse; use different assessments techniques to promote students reasoning and reflect their teaching practices in making instructional decisions; apply a variety of instructional techniques such as collaborative learning, student-centre, and inductive and deductive reasoning methods of teaching; connect with the past experiences to develop the new knowledge; help students to explore the mathematical concepts by multiple representations supporting them graphically or numerically, and verify them algebraically or geometrically; and support students to judge the validity of mathematical arguments and draw appropriate conclusions. In open-ended tasks teachers should allocate time to support students to reason without any constraints to produce correct or incorrect answers to predetermined outcomes. Mathematics teaching should be geared towards supporting learners to express their reasoning. Within the mathematics reasoning Discourse, learners are required to provide justification for any response they give to a problem. However, majority of textbooks are designed to teach students particular mathematical techniques and procedures rather than to help students develop thinking skills necessary for the learners to take part in the mathematical reasoning Discourse.

Based on the findings of the study, the recommendations were as follows: Training should be given to teachers how to enhance the students' reasoning skills. The curriculum developers, implementers and educational evaluators and school heads should enforce teachers apply teaching methods and assessment techniques that enhance reasoning skills into the teaching of mathematics. Finally, Critical Thinking should be made compulsory and integrated in all primary and secondary school curriculum, scheme of work, lesson note, lesson plan and in the classroom when teaching and learning take place because the concepts serves as learning and teaching aids and also makes the students understand the concept better. Mathematics textbooks should be designed to enable learners communicate their mathematical reasoning. Asking students open ended questions and questions that require learners to justify and give explanations to their answers should be included in the textbook. The teacher should also ask learners questions that require learners to communicate their mathematical reasoning.

COMPETING INTERESTS

Author has declared that no competing interests exist.

REFERENCES

1. Reston VA. Principles and standards for school mathematics.NCTM; 2000.
2. MoE. Grade 1-4 mathematics syllabus; Addis Ababa; Ethiopia; 2008.
3. MoE. Grade 5-8 mathematics syllabus; Addis Ababa; Ethiopia; 2008.
4. Kusumah YS. Logika matematika elementer. Bandung: Tarsito; 1986.

5. Alamsyah. Suatu pembelajaran untuk meningkatkan kemampuan penalaran analogi matematik. Tesis magister pada PPs UPI bandung: Tidak Diterbitkan; 2000.
6. Dahlan JA. Meningkatkan kemampuan penalaran dan pemahaman matematik siswa sekolah lanjutan tingkat pertama me lalui pendekatan open-ended. Disertasi pada PPs UPI. bandung: Tidak Dipublika Sikan; 2004.
7. Matlin MW. Cognition (third edition). New York: Harcourt Brace Publisher; 1994.
8. Kadir Afandi MF. Enhancing the students' ability of reasoning on aspect to formulate the counter example through problem solving strategies make an organized list; Proceeding of international conference on research, implementation and education of mathematics and sciences, Yogyakarta State University. 2014;18-20.
9. Shield M, Galbraith P. The analysis of student expository writing in mathematics. Educational Studies in Mathematics. 1998;36(1):29-52.
10. Albert LR. Outside-in inside-out: Seventh grade students' mathematical thought processes. Educational Studies in Mathematics. 2000;41(2):109-141.
11. Klymchuk S. Riddles with money: Mathematical reasoning development and lateral thinking (in Spanish). Mexico: Editorial Trillas, Mexico; 2008.
12. Mevarech ZR, Kramarski B. Improve: A multidimensional method for teaching mathematics in heterogeneous classrooms. American Educational Research Journal. 1997;34:365–394.
13. Lester FK, Garofalo J, Kroll DL. The role of metacognition in mathematical problem solving: A study of two grade-seven classes. ERIC. 1989;1–126. (ERIC Document Reproduction Service No. ED314255).
14. Polya G. How to solve it? Garden City, NY: Doubleday; 1945.
15. Mandelbrot B. Fractals the computer, and mathematics education. In proceedings of the 7th international congress on mathematical education, edited by Claude Gaulin, et al. 1992;77-98.
16. National Council of Teachers of Mathematics, curriculum and evaluation standards for school mathematics. Reston VA: National Council of Teachers of Mathematics; 1989.
17. Cooley L. Writing in calculus and reflective abstraction. Journal of Mathematical Behavior. 2002;21:255-282.
18. Campbell AE, Adams VM, Davis GE. Cognitive demands and second language learners: A framework for analyzing mathematical instructional contexts. Mathematical Thinking and Learning. 2007;9(1):3-30.
19. Doerr HM, English LD. Middle grade teachers' learning through students' engagement with modeling tasks. Journal of Mathematics Teacher Education. 2006;9(1):5-32.
20. Francisco JM, Maher CA. Conditions for promoting reasoning in problem solving: Insights from a longitudinal study. Journal of Mathematical Behavior. 2005;24:361-372.
21. Goos M. Learning mathematics in a classroom community of inquiry; Journal for Research in Mathematics Education. 2004;35(4):258-291.
22. Maher CA, Martino AM. The development of the idea of mathematical proof: A 5-year study. In F. Lester (Ed.). Journal for Research in Mathematics Education. 1996;27(2):194-214.
23. Maher CA. Children's reasoning: Discovering the idea of mathematical proof. In Blanton M, Stylianou D (Eds.), teaching and learning proof across the grades. New Jersey: Taylor Francis – Routledge; 2009.
24. Mollborn S, Hoekstra A. A meeting of minds: Using clickers for critical thinking and discussion in large sociology classes. Teaching Sociology. 2010;38(1):18-27.
25. Mansoor F, Pezeshki M. Manipulating critical thinking skills in test taking. International Journal of Education. 2012;4(1):153-160.
26. Facione PA. Critical thinking: A statement of expert consensus for purposes of educational assessment and instruction. Millbrae CA: The California Academic Press; 1990.
27. Moyer PS, Milewicz E. Learning to question: Categories of questioning used by pre-service teachers during diagnostic mathematical interviews. Journal of Mathematics Teacher Education. 2002;5:293-315.
28. Martino AM, Maher CA. Teacher questioning to promote justification and generalization in mathematics: What

research practice has taught us. Journal of Mathematical Behavior. 1999;18(1):53-78.
29. Sahin A, Kulm G. Sixth grade mathematics teachers' intentions and use of probing, guiding, and factual questions. Journal of Mathematics Teacher Education. 2008;11(3):221-241.
30. Slavin RE. Research on cooperative learning and achievement: What we know, what we need to know. Contemporary Educational Psychology. 1996;21:43–69.
31. Schoenfeld AH. Mathematical problem solving. Orlando FL: Academic Press; 1985.

Peer-review history:
The peer review history for this paper can be accessed here:
http://sciencedomain.org/review-history/17609