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On a Class of Universal Probability Spaces: Case of Complex Fields

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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ABSTRACT

The objective of this paper is to extend the Universal Probability Space (UPS) in [1] to include complex events. The UPS consists of Borel sets, elements of which are tensors. It is shown that the UPS has a defined metric and this metric is in fact the probability measure (P). The metric as a probability measure is proven to exist for any tensor event ($x \in \mathbb{R}^d$) in the space of all tensor fields, ($\mathbb{R}^d$). In this paper it is shown that for any complex event, ($x \in \mathbb{C}^d$) in a space of all complex tensor fields, ($\mathbb{C}^d$), a probability measure (P) in the form of a metric exists. To this effect several theorems are introduced and proven, mainly by modifying concepts introduced in [2], [3], [4], [5], to include complex fields. Finally following [6], [7], [8], a case is demonstrated in order to compare probability as a metric for complex events with classical probability. The objective of the case study is to show that metric probability is a more realistic measure than classical probability for complex events.

Keywords: Universal Probability Space; Borel tensor sets; Borel tensor field; complete tensor space; metric; probability measures.

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1 INTRODUCTION

Classical probability gives an abstract estimate of an occurrence of an event. The main trust of estimation is the number of times an outcome is observed in (n) trials. This estimation does not take into account the environment in which an event occurs, or the traits and characteristics inherent to the event itself that cause the event to happen. In other words, the causality of the occurrence of an event is systematically ignored when calculating probabilities. Though classical probability gives us a peak at the possibility of an occurrence of an event it can not give us the true possibility of an occurrence of an event. An event is considered to be some transformation based on variables that constitute the environment of such an event. The traits and characteristics internal to it constitute the causality of the event. A good example of this is given by [9] where in 1999 a British solicitor was convicted of murdering her two children based on the testimony of an statistician who had estimated the probability of two children dying in the same family to be as small as 1 in 73 million. Later on the discovery of the sudden infant death syndrome proved the probability to be wrong due to bad statistics and ignoring causality. To be able to get a true probability an event should be formulated as a tensor. A tensor is a mathematical formulation of cause and effect.

Probability measure in the case where an event is described by a tensor can not be the same as the classical probability calculation. To measure the impact of causality in the occurrence of a tensor event point, the metric probability is introduced. Metric probability gives the probability of the occurrence of a tensor event point, as the ratio of the distance between each casual factor in (d) dimension ($\Re^d$) or the metric, and the general metric given all causal factors in ($\Re^d$). Formulation of such a probability metric is given in section (3).

To construct a metric probability, it is shown that it is possible to have a probability space where an event can occur if it is in a Borel field ($\boldsymbol{B}$) that contains a group of finite Borel sets that are mappings in a tensor space (Ω), i.e. ($\boldsymbol{B} = \bigcup_{l=1}^{M} B_l^i$) and ($\boldsymbol{B} = \prod_{l=1}^{M} B_l^i$), the superscript ($i$) represents the tensor indexing of coordinates, and (l) represents the number of Borel tensor sets contained in a tensor field ($\boldsymbol{B}$). Each Borel tensor set can be identified as ($B^i \in \Omega : B^i = B_{j_s}^{i_r} \otimes e_{i_1,\ldots,i_r}^{j_1,\ldots,j_s}$), where ($e_{i_1,\ldots,i_r}^{j_1,\ldots,j_s}$) constitutes the basis in the tensor space (Ω). The advantage of each Borel set to be a mapping in a tensor space is that it acquires the following properties; (a) closure, the product of every two mappings of the Borel set (B) is a mapping of the Borel set; (b) Inverse: for every mapping of the Borel set, there is an inverse mapping that is in the Borel set. The class of such alternative probability space is called the Universal Probability Space (UPS), [1]. To sum up, the UPS consists of Borel sets, elements of which are mappings or tensors. UPS represents a more general probability space.

Although big data allows for the discovery of causes behind an occurrence of an event, it is not always possible to draw the right inferences. This could be due to many factors. For example, not all variables that can cause an event can be formulated accurately, or the actual relationship between the advent of an event and its cause is only approximately identified. In this case, the best representation of causality is to formulate an event as a complex tensor. The objective of this paper is to introduce a metric probability for the case of complex tensors. In order to include a class of tensor fields in the (UPS), it is necessary to find a representation of complex tensor disks in the (Ω) space for the metric probability such that it includes transformation sets of singularities. This can be done using germ sets. Let's assume that there are 2 complex tensor disks (U), and (V) in some complex tensor space ($U \subset \mathbb{C}^d$), and ($V \subset \mathbb{C}^d$). Let ($\boldsymbol{z}$) be a set of tensor points in the complex tensor space ($U \subset \mathbb{C}^d$), then the germ of a transformation tensor ($\boldsymbol{w}$) at ($\boldsymbol{z}$) is the set of all tensor points ($\boldsymbol{w}, V$), ($\boldsymbol{z} \in U$), where (V) is a complex tensor disk subset of the complex tensor space ($\mathbb{C}^d$), ($V \subset \mathbb{C}^d$), where ($\boldsymbol{w} = \boldsymbol{z}$) on ($U \cap V$). A germ set can be denoted as ($[\boldsymbol{w}]\boldsymbol{z}$), [10].

It must be shown that an event can occur in a complex tensor field ($\boldsymbol{U}$) that contains a group of finite complex tensor disks ($\boldsymbol{U} = \bigcup_{l=1}^{M} U_l^i$) and ($\boldsymbol{U} = \prod_{l=1}^{M} U_l^i$). The superscript ($i$) represents the tensor indexing of coordinates, and (l) represents the number of complex tensor disks contained in a complex tensor field ($\boldsymbol{U}$). Suppose that there are (n) finite number of open complex tensor disks $(U_1, U_2, ..., U_n)$.

It is assumed that the pairwise intersections of $(U_l \cap U_k)$, $(l \geq 1, k \leq n)$ and the union of these open complex tensor disks $(U_1 \cup U_2 \cup .... \cup U_n \neq 0)$ are non- empty complex tensor sets. In order to have non-empty complex tensor disks, it must be shown that all singularities can be transformed into germ sets or composition of germ sets. Singularities are points that are not included in complex tensor disks. One such point is the point (zero). Complex tensor disks contain metrics, [10]. A metric in complex disks, is a parametrized metric. Let $(\eta(t))$ be parametrized metric defined by a distance between two complex sets that are defined based on parameter (t). (t) is defined in interval $(a \leq t \leq b)$, where $(a, b \in \mathbb{R})$. There exists a union of open disks of complex sets with a covering, $(U_1 \cup U_2 \cup .... \cup U_n \neq 0)$ that contains the metric. The parametrized metric, $(\eta(t))$ includes singularities if it is closed meaning is a covering. A covering implies that the complex disks are closed sets. Equivalent metric in the UPS can exist as a transformation from a complex tensor space. A metric is transformed into the UPS space, $(\Omega, \boldsymbol{B}, P)$, if it is transformed into the distance between two tensor germ sets. Metric parametrization is not necessary in a tensor context. It is shown that the metric of tensor germ sets is a natural covering. The details of a metric of germ sets in the UPS are given in the next section.

2 THE UNIVERSAL PROBABILITY SPACE, UPS: SOME BASIC DEFINITIONS ON COMPLEX CASE

A complete space consisting of Borel tensor fields $(\Omega,\boldsymbol{B})$ and a probability measure (P), $(\Omega, \boldsymbol{B}, P)$ is the UPS. This is proven in 2.4 and 2.5. The proofs are provided in [1].

Theorem 2.1. *If (Ω) is a tensor space, where a class of Borel tensor fields $(\boldsymbol{B})$ can be defined, then the ensemble $(\Omega,\boldsymbol{B})$ constitutes a complete space.*

Theorem 2.2. *Given that $(\Omega,\boldsymbol{B})$ is a complete space, then there is a metric on the tensor field $(\boldsymbol{B})$, [2], [3], [4], [5]. This metric is the probability measure (P). The complete space $(\Omega,\boldsymbol{B})$ with the probability measure (P) is the UPS.*

Definition 2.1. Let (s) be a complex tensor set of singularities such that $(s = \{s_j\}; j = 1, ..., m)$ is in a complex space, $(s \in \mathbb{C}^d)$. Let any transformation of the singularities be defined as $(\bar{s} = \{\sum_j \bar{\lambda_{ij}} s_j\}; j = 1, ..., m)$ such that $(\bar{s} \in V)$, $(V \subset \mathbb{C}^d)$ is defined $((\bar{s}, V) \subset \mathbb{C}^d)$. The matrix (λ_{ij}) is an $(i \times j)$ transformation matrix. The linear transformation is a tensor germ set denoted by $([\bar{s}]_s)$. Let there exist another transformation given by $(\tilde{s} = \{\sum_j \tilde{\lambda_{ij}} s_j\}; j = 1, ..., m)$ such that $(\tilde{s} \in U)$ and $(U \subset \mathbb{C}^d)$, $((\tilde{s}, U) \subset \mathbb{C}^d)$. This transformation is a tensor germ set denoted by $([\tilde{s}]_s)$. If the two tensor germ sets $([\bar{s}]_s = [\tilde{s}]_s)$ are equal in the intersection of the two regions $(V \bigcap U)$, then the tensor germ set $([\bar{s}]_s)$ is the tensor germ set of the singularities (s).

Definition 2.2. Let there exists two subregions $(V \subset \mathbb{C}^d)$ and $(U \subset \mathbb{C}^d)$. Let $(([\bar{s}]_s, [\tilde{s}]_s) \in (V \bigcap U))$, then any composition of the two tensor germ sets, $(([\bar{s}]_s))$ and $(([\tilde{s}]_s))$ is denoted by $(\hat{s} = ([\bar{s}]_s \circ [\tilde{s}]_s)))$. The composition tensor germ set $(\hat{s} \in (V \bigcap U))$ is the tensor germ set of the singularities (s).

Definition 2.3. Let $(z \in \mathbb{C}^d)$ and $(z' \in \mathbb{C}^d)$ be two complex tensor points. A metric can be defined as $(dz =\mid z - z' \mid= \sqrt{(z - z')^2})$, where $(dz \in \mathbb{R})$. Such a metric is always a transformation from a complex tensor space $(\mathbb{C}^d)$ to a real space $(\mathbb{R})$. A metric for a complex tensor set (s) is a metric of the tensor germ sets of the complex tensor set (s). Given two tensor germ sets $(([\bar{s}]_s))$ and $(([\hat{s}]_s))$, such that $(([\bar{s}]_s, [\hat{s}]_s) \in (V \bigcap U))$, a metric is defined as $(ds =\mid [\bar{s}]_s - [\hat{s}]_s \mid= \sqrt{([\bar{s}]_s - [\hat{s}]_s)^2})$. The metric $(ds \in \mathbb{R})$ is in a real space $(\mathbb{R})$.

Corollary 2.3. *The UPS,$(\Omega, \boldsymbol{B}, P)$ contains a class of complex tensor fields $(\boldsymbol{A} \subset \Omega)$, elements of which consist of tensor germ sets or a composition of tensor germ sets.*

Proof. By definition 2.1, the class of tensor fields ($\boldsymbol{A}$) must contain any two complex tensor sets ($V \subset \mathbb{C}^d$) and ($U \subset \mathbb{C}^d$) in such a way that ($\boldsymbol{A}$) contains both the intersection of the two complex tensor sets ($V \bigcap U$) and the union of the of the two complex tensor sets ($V \bigcup U$). By definition 2.1, since ($\boldsymbol{A}$) contains ($V \bigcap U$), then the intersection contains tensor germ sets. □

Theorem 2.4. *Given that (Ω,$\boldsymbol{B} \cup \boldsymbol{A}$) is a UPS, then there exists a metric that is a covering.*

Proof. A metric is a covering if it contains singularities. By definition 2.1 any complex tensor singularity ($s \in U$) can be represented by a complex tensor germ set ($[\bar{s}]_s \in (V \bigcap U)$). Any metric containing complex tensor germ set ($[\bar{s}]_s$) is continuous, in such a way that the derivative of the complex tensor germ set is non-zero, ($[\bar{s}]'_s \neq 0$). Therefore, the inverse complex tensor germ set ($[\bar{s}]_s^{-1}$) can be defined. Given that all the prerequisites of a covering are satisfied, then the intersection of the two complex tensor sets ($V \bigcap U$) is a covering. □

Theorem 2.5. *Given that (Ω,$\boldsymbol{B} \cup \boldsymbol{A}$) is the UPS, then there exists a metric on the tensor field ($\boldsymbol{A} \cap \boldsymbol{B}$). This metric is the probability measure (P).*

Proof. By definition 2.3, and by Corollary 2.3, the complex tensor field ($\boldsymbol{A}$) contains tensor germ sets and its metric. This metric is defined as ($ds = \mid [\bar{s}]_s - [\hat{s}]_s \mid = \sqrt{([\bar{s}]_s - [\tilde{s}]_s)^2}$). By definition 2.1, ($[\bar{s}]_s$) and ($[\hat{s}]_s$) are defined as ($[\bar{s}]_s = \{\sum_j \bar{\lambda_{ij}} s_j\}$, and ($[\hat{s}]_s = \{\sum_j \hat{\lambda_{ij}} s_j\}$). For a metric to be on the tensor field ($\boldsymbol{A} \cap \boldsymbol{B}$), the transformation matrices ($\bar{\lambda_{ij}}$), and ($\hat{\lambda_{ij}}$) must be in the Borel tensor field ($\boldsymbol{B}$). □

Corollary 2.6. *Given that (Ω,$\boldsymbol{B} \cup \boldsymbol{A}$) is the UPS, then any Borel tensor set (B^i) contains transformation matrices ($\bar{\lambda_{ij}}$) .*

Proof. Let's define the transformation matrix ($\bar{\lambda_{ij}}$) as the measure of variations from one coordinate sytem to another denoted as ($\bar{\lambda_{ij}} = (\frac{\partial \bar{s}^i}{\partial \check{s}^j})$), where ($[\check{s}]_s$) is a tensor germ set different from ($[\bar{s}]_s$), ($[\check{s}]_s \neq [\bar{s}]_s$). By Theorem 2.4, the intersection of any two complex tensor sets ($V \subset \mathbb{C}^d$) and ($U \subset \mathbb{C}^d$), ($V \bigcap U$) must be a covering. The existance of a covering allows the application of ($Lindel\ddot{o}f's$) Lemma which states that any tensor germ set ($[\bar{s}]_s$) has a limit, ($limsup_{s \to \xi} \mid [\bar{s}]_s \mid < m$). ($\xi$) is on the boundary of ($V \bigcap U$). ($m > 0$) is an arbitrary positive integer. Replacing the tensor germ set with its equivalent, ($limsup_{s \to \xi} \mid \sum_j \bar{\lambda_{ij}} s_j \mid < m$), follows that the transformation matrix ($\bar{\lambda_{ij}}$) is bounded. The most obvious bound is any Borel tensor set (B^i). Therefore, the transformation matrix is in any Borel tensor set (B^i), ($\bar{\lambda_{ij}} \in B^i$) □

3 EXAMPLES OF METRIC PROBABILITIES: CASE OF 3 AND 6 FACTORS

In this section examples of 3 and 6 causal factors metric probabilities are given below. Consider, the case of a toss of a coin. In classical probability, both outcomes, "head" or "tail" have the same probability of one half. This is the case because the two outcomes are considered independent of each other. No other factors as causes of the outcomes are considered. Now, let's assume that the outcome of interest is "head". Assume that to get this outcome, (n) factors can be considered as causes. Metric probability for an (n) factors tensor event is calculated by following the steps given hereafter. Let a tensor event point be designated by ($\bar{\mathbf{X}} \in \mathbf{\Omega}$) in the UPS. The tensor event point ($\bar{\mathbf{X}}$) is formulated as a transformation from one coordinate system ($\mathbf{x} = (x_1, ..., x_n)$) which represents the vector of factors either internal or environmental related to the tensor event point ($\bar{\mathbf{X}}$) to another coordinate system ($\bar{\mathbf{x}} = (\bar{x}_1, ..., \bar{x}_n)$) which represents variations in the measurements of causal factors due to random occurrences, [11]. This is expressed as ($\bar{\mathbf{X}} = \mathbf{A} \otimes \mathbf{X}$) or in the expanded form ($\bar{x_i} = (a_{ij} x_j), i = 1, ..., n; j = 1, ..., n$). The coefficient Matrix ($\mathbf{A} = \{a_{ij}\}$) consists of the derivatives of ($\bar{x}$) with respect to (x). This is denoted as ($\mathbf{A} = \{a_{ij}\} = (\frac{\partial \bar{x}^i}{\partial x^j})$). Subscripts (j) correspond to the causal factors. Let matrix ($\mathbf{G}$) be a positive definite matrix given as ($\mathbf{G} = (\mathbf{A}^T \mathbf{A})$). The diagonal entries of matrix ($\mathbf{G}$), ($(\mathbf{A}^T \mathbf{A}) = (a_{ij})^2, i = j$) constitute the elements of the general metric $((ds))$. The general metric $((ds))$ is formulated as $((ds) = (a_{11})^2 + (a_{22})^2 + ... + (a_{nn})^2)$. The characteristic equation is given as $(\mathbf{\Lambda} = \mid \mathbf{G} - \boldsymbol{\lambda}\mathbf{I} \mid = 0)$, where matrix $(\mathbf{I})$ is an identity matrix. The eigenvalues $(\boldsymbol{\lambda} = (\lambda_1, ..., \lambda_n))$ are ranked from the lowest to the highest $((\lambda_1 < \lambda_2 < ... < \lambda_n))$. The eigen vectors corresponding to these eigenvalues are denoted by $(\mathbf{V} = \{v_i\})$, [12], [13]. The estimated event point $(\hat{\mathbf{X}})$ is formulated as $(\hat{\mathbf{X}} = \boldsymbol{\lambda}\mathbf{V})$ or in an expanded form $(\hat{x}_i = \lambda_i \times v_i)$. The calculated distance between each eigenvector $(\{v_i\})$ and each estimated event point $(\hat{x}_i)$ is given as $(ds(v_i, \hat{x}_i))$, [14]. The estimated distance is calculated as $(ds(v_i, \hat{x}_i) = \sqrt{v_i - \lambda_i \times v_i})$. The metric probability for each causal factor (i), (P_i) is equal to the ratio of the calculated metric for each causal element, $(ds(v_i, \hat{x}_i))$ to the square root of the general metric $((ds))$, given as $(P_i = \frac{ds(v_i, \hat{x}_i)}{\sqrt{ds}})$. If the estimated distance for factor (i) is close to the general metric, then the causal factor corresponding to the estimated distance for factor (i) is the significant cause in getting the desired outcome.

Let's calculate the metric probability of getting a "head" in a toss of a coin, when (3) causal factors are identified. These factors are: 1) weight of the coin, 2) distance of a toss, 3) physical condition of the person tossing the coin (shape of hands, trembling or steady hands). The event $(\bar{\mathbf{X}} = (\bar{x}_1, \bar{x}_2, \bar{x}_3) \in (\Omega, \boldsymbol{B}, P))$ consists of getting the outcome "head" as a function of the weight of the coin, $(\bar{x}_1)$, getting the outcome "head" as a function of the distance of a toss $(\bar{x}_2)$, and getting the outcome (head) as a function of the physical condition of the person doing the tossing, $(\bar{x}_3)$. The vector of causal factors $(\mathbf{X} = (x_1, x_2, x_3) \in (\Omega, \boldsymbol{B}, P))$ represents the (3) causal factors, weight, (x_1), distance of a toss (x_2), and physical conditions, (x_3). It is assumed that the coefficient matrix $(\mathbf{A})$ is given through observation. In this case the coefficient matrix $(\mathbf{A})$ is a (3×3) matrix. The number of rows and columns represents the (3) causal factors, weight, distance, and physical condition. The entries of matrix $(\mathbf{A})$ are given as $(\mathbf{A} = \{a_{ij}\} = (\frac{\partial \bar{x}^i}{\partial x^j}))$. $((\frac{\partial \bar{x}^i}{\partial x^j}))$ is the variations in causal factors measurements due to random occurrences. The term $(\frac{\partial \bar{x}^i}{\partial x^j})$ is taken to be approximately equal to variations in each causal factor. For example if weight is graded as (Low, Medium, Heavy), and tossing distance as (Short, Medium, Far), and physical condition as (Shape of hands, shaky hands, and Steady hands), then the variations let's say in the weight factor signifies general observed variations in the weight of a generic coin. The same definition applies for the other causal factors. This variation is coded as a quantity between (0,1). Based on matrix $(\mathbf{A})$, the Eigenvalues and the eigenvectors of the tensor event point $(\bar{\mathbf{X}})$ can be found. The metric calculated given the observed matrix $(\mathbf{A})$, is $((ds) = 1.1218977)$. A metric probability for each of the (3) causal factors is calculated to be $(P_1 = 0.8500716)$ for the weight factor, (x_1), $(P_2 = 0.6051443)$ for the distance of a toss factor, (x_2), and $(P_3 = 1.9245483)$ for physical conditions factor, (x_3). Causal factors within the general metric are factors (x_1), and (x_2) the weight of the coin, and the distance of the toss respectively since they are within the general metric (ds). Therefore, the existence of causal factors (x_1), and (x_2) creates the desired effect "head". But since the causal factor (x_1) is closer to the general metric, this factor is the major causal factor in getting the outcome "head". Figure 1, gives a visual example of eigenvalues for the (3) causal factors case. Eigenvalues in this context represent real values of the causal factors. In 3D representation, the z-axis designates the values of the causal factors. The x-axis designates the number of causal factors, and the y-axis is the scale used for 3D demonstration.

Figure 2, gives an example of eigenvectors. Eigenvectors represent the evolution of each eigenvalue. The first two causal factors, the weight of the coin (in blue), and the distance of the toss (in green) have unstable eigenvectors, while the third causal factor, the physical condition of the person (in red) has a stable eigenvector.

Figure 3, gives an example of eigenvectors field. The length of the arrows is proportional to the intensity of the field. The eigenvectors field confirms the conclusion that the first two causal factors are unstable.

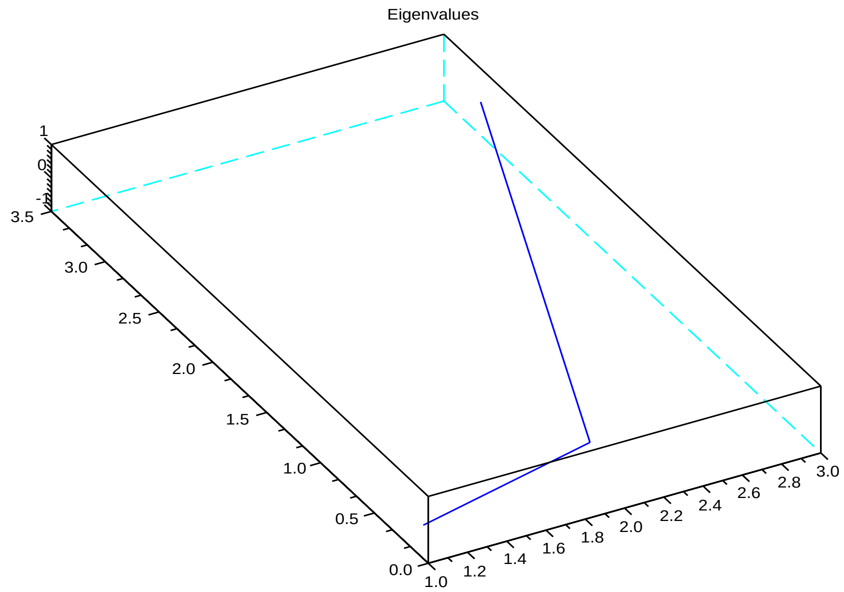

Fig. 1. Example of eigenvalues for a (3) factors case

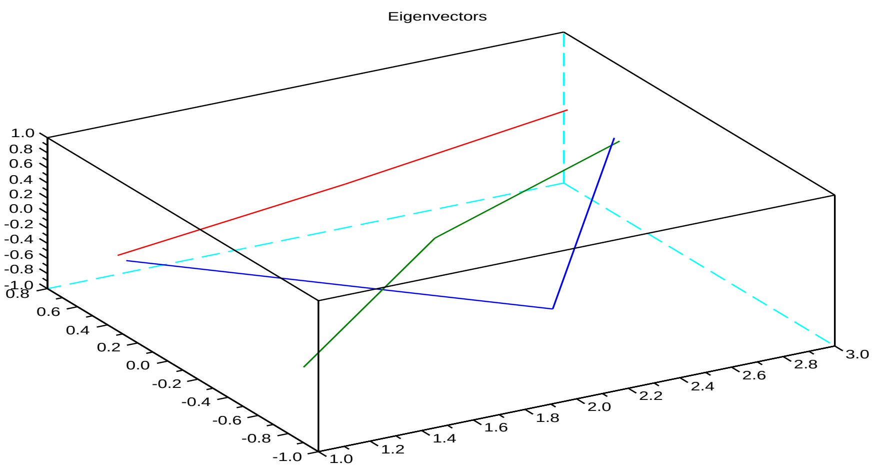

Fig. 2. Example of eigenvectors for a (3) factors case

Now consider the case where the outcome "head" of a toss of a coin depends on (6) causal factors in ($\in \ (\Omega, \boldsymbol{B}, P)$), ($\mathbf{X} = (x_1, x_2, x_3, x_4, x_5, x_6) \in (\Omega, \boldsymbol{B}, P)$). Let these factors be 1) weight of the coin, (x_1), 2) distance of the toss, (x_2), presence of others, (x_3), physical condition of the person tossing the coin (shape of hands, trembling hands, steady hands), (x_4), mental condition of the person tossing the coin (happy, sad, Zen, nervous), (x_5), and finally, air quality, (x_6). Given matrix ($\mathbf{A}$), of size (6×6) corresponding to variations in the (6) causal factors, the corresponding metric can be calculated to be ($(ds) = 1.2$). Given the characteristic function, eigenvalues, eigenvectors, and calculated metric are obtained. Metric probability of each factor is calculated to be, ($P_1 = 1.2$) for the weight factor, (x_1), ($P_2 = 1.2$) for the distance of a toss factor, (x_2), ($P_3 = 1.2$) for presence of others factor, (x_3), ($P_4 = 1.1$) for physical conditions factor, (x_4), ($P_5 = 1.0$) for mental condition factor, (x_5), and ($P_6 = 1.0$), for the air quality factor (x_6). Factors (x_5), the mental condition of the person tossing the coin, and (x_6), the air quality are within the acceptable metric. They are the most likely causes of the desired outcome "head". Figure 4, gives a visual example of eigenvalues for the (6) causal factors case. In 2D representation, the x-axis designates the number of causal factors. The y-axis designates the values of the causal factors. Figure 5, gives an example of eigenvectors, and Figure 6, gives an example of eigenvectors field. In this particular trial, all eigenvectors are stable. This observation is confirmed by the eigenvectors field. Since the eigenvalues of causal factors (5), and (6) have the highest values, and all the eigenvectors are stable and the eigenvectors field has the same intensity, it is reasonable that factors (5), and (6) are the most likely causes of the outcome event "heads".

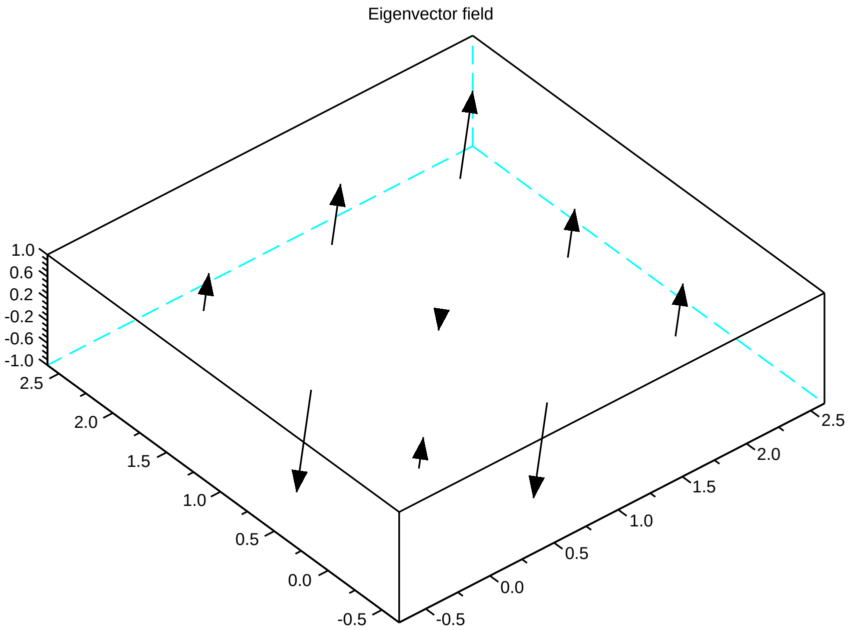

Fig. 3. Example of eigenvectors field for a (3) factors case

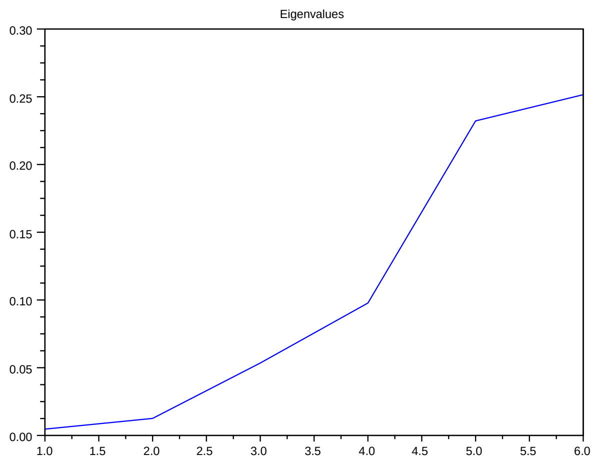

Fig. 4. Example of eigenvalues for a (6) factors case

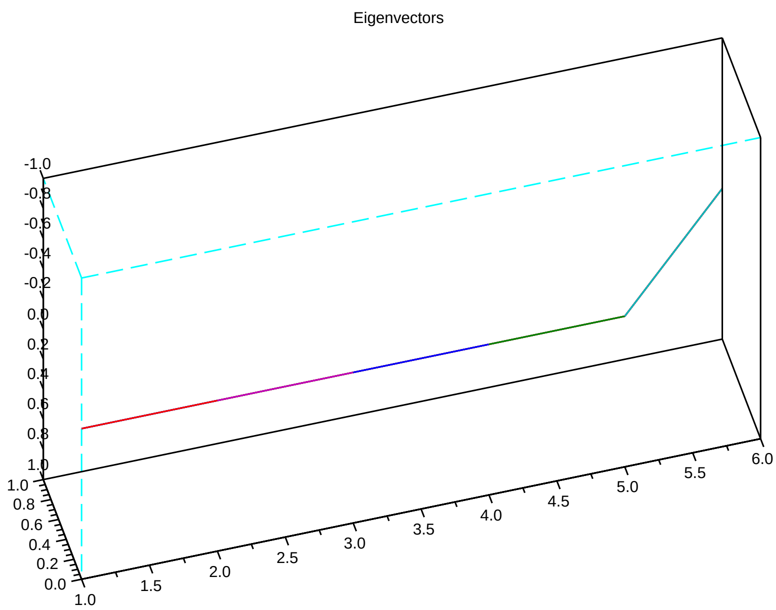

Fig. 5. Example of eigenvectors for a (6) factors case

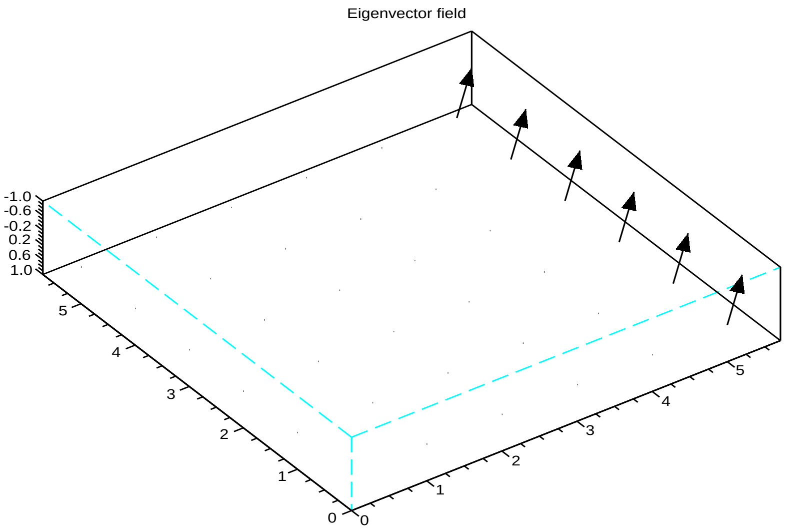

Fig. 6. Example of eigenvectors field for a (6) factors case

In case, there are uncertainties in either observations or the identification of the degrees of causal factors, it is possible to formulate events and their causal elements as complex tensors in complex tensor space, ($\bar{\mathbf{X}} \in \mathbb{C}^d$), and ($\mathbf{X} \in \mathbb{C}^d$). Naturally, matrix ($\mathbf{A}$), and the characteristic function ($\mathbf{\Lambda}$), the eigenvalues and eigenvectors all remain in the complex tensor space, ($\mathbf{A} \in \mathbb{C}^d$), ($\mathbf{\Lambda} \in \mathbb{C}^d$), and ($\mathbf{V} \in \mathbb{C}^d$). As is discussed in section (2), the metric is in UPS, ($(ds) \in (\Omega, \boldsymbol{B}, P)$). As is discussed in the introduction and section (2), the calculated metric for each of the causal factors (x_i) is also in the UPS ($ds(v_i, \hat{x}_i) \in (\Omega, \boldsymbol{B}, P)$). Let's revisit the (6) causal factors case. The difference this time is that each of the entries of the required matrix ($\mathbf{A} \in \mathbb{C}^d$), is a transformation in a complex tensor space of any observed point, ($x_i = x_i(w_i, z_i) \mid w \in \Re^d, z \in \mathbb{C}^d$)) into the space of analytical functions or complex planes, ($x_i(w_i, z_i) \longrightarrow f(x_i(w_i, z_i))$). The metric is calculated to be equal to ($(ds) = 1.9789175$). The metric probability for each of the (6) causal factors is: ($P_1 = 1.738227$) for the weight factor, (x_1), ($P_2 = 0.7566326$) for the distance of a toss factor, (x_2), ($P_3 = 0.4131377$) for presence of others factor, (x_3), ($P_4 = 0.4437326$) for physical conditions factor, (x_4), ($P_5 = 0.3692848$) for mental condition factor, (x_5), and ($P_6 = 0.2004571$), for the air quality factor (x_6). Causal factor (x_2), distance of toss is closest in value to the general metric. Therefore, the presence of casual factor (x_2), results in outcome "head". Figure 7, gives a visual example of complex valued eigenvalues for the (6) causal factors case. The x-axis represents the real part, and the y-xis represents the imaginary part. Only those causal factors with low magnitude of imaginary part are stable enough to be considered as plausible significant causal factors. Figure 8, gives an example of complex valued eigenvectors. Eigenvectors show that all (6) factors are stable in general and could potentially be significant in getting outcome "head". In fact, eigenvectors show that the imaginary part stays smaller than the real part which signifies stability. Figure 9, gives an example of complex eigenvectors field. The eigenvectors field shows high intensity and movement towards stability.

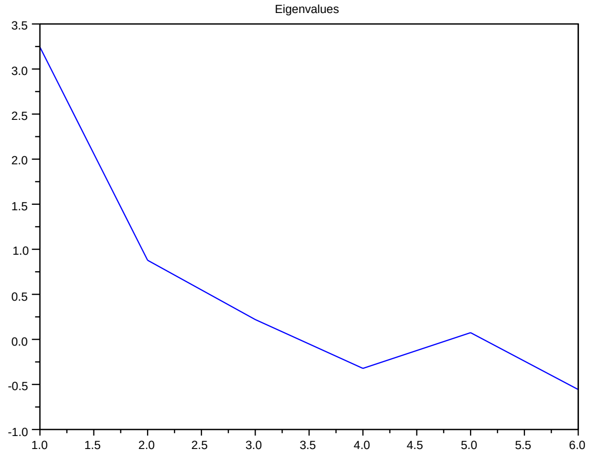

Fig. 7. Example of complex valued eigenvalues for a (6) factors case

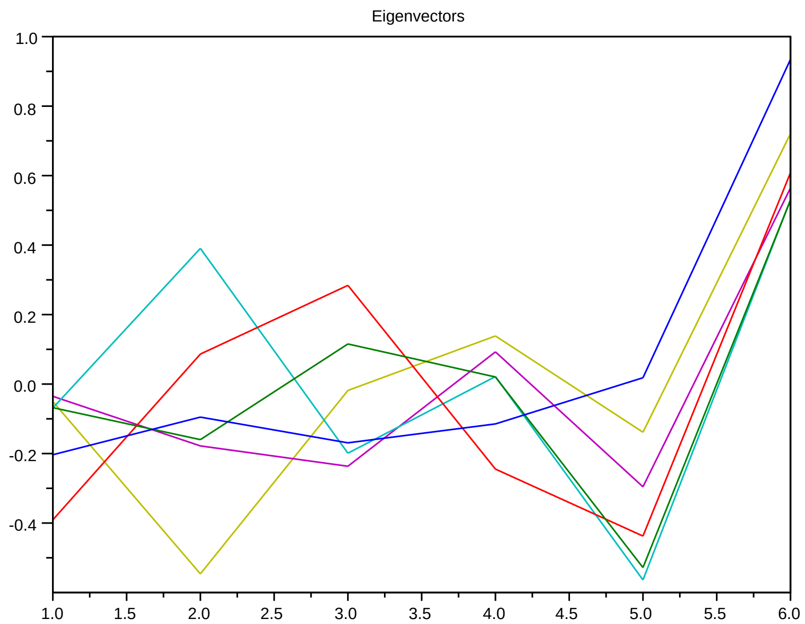

Fig. 8. Example of complex valued eigenvectors for a (6) factors case

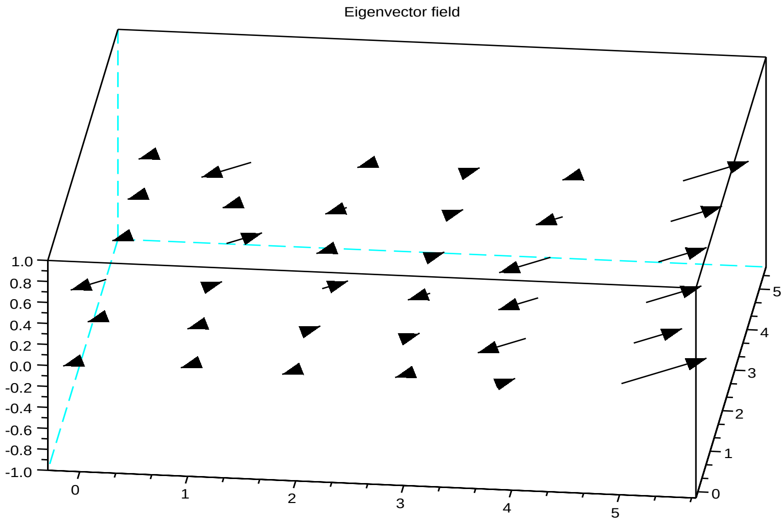

Fig. 9. Example of complex valued eigenvectors field for a (6) factors case

4 CONCLUSION

Metric robability provides a reasonable alternative to the standard probability calculation. This is the case because it takes into account 1) All factors internal to an occurrence of a tensor event point, 2) plus, all factors that represent the environment in which an event occurs. An event is defined as an effect of several causal elements. Thus, the calculation of the probability of an occurrence of the tensor event point is not limited to an abstract representation, but a more realistic calculation that is analytic. In the case of complex tensors, metric probability of the occurrence of a complex tensor event, has the following advantages: 1) probabilities can be calculated for events in the UPS, $((\Omega, \boldsymbol{B}, P))$. 2) The probability is based on metric calculation which is more reliable than the number of trials used in classical probability calculation. 3) Metric probability has applications in many areas where a simple approximation based on a limited number of trials is not sufficient.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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