

3(2): 48-61, 2021; Article no.IAARJ.70213

J. J. Rawal1* and Nishant Malhotra2

1 The Indian Planetary Society, B-201, Vishnu Apartments, L.T. Road, Borivali (W), Mumbai – 400092, Maharashtra, India. ² Mechanical Engineering Division, Thakur College of Engineering and Technology, Thakur Village, Kandivali (East), Mumbai – 400101, Maharashtra, India.

Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

Article Information

Editor(s): (1) Dr. Magdy Rabie Soliman Sanad, National Research Institute of Astronomy and Geophysics, Egypt. (2) Dr. David Garrison, University of Houston-Clear Lake, USA. (3) Dr. Hadia Hassan Selim, National Research Institute of Astronomy and Geophysics, Egypt. *Reviewers:* (1) Himanshu Dehra, American Institute-Industry Forum on Energy, Canada. (2) Dmitriy Beznosko, Clayton State University, Georgia. (3) Anil Kumar Koneti, Tirumala Engineering College, India. (4) Giorgio Sonnino, Universite' Libre de Bruxelles (ULB), Belgium. (5) Sudeshna Mukerji, Jadavpur University, India. Complete Peer review History: https://www.sdiarticle4.com/review-history/70213

> *Received 14 May 2021 Accepted 19 July 2021 Published 28 July 2021*

Original Research Article

ABSTRACT

Rawal [1 to 11] studied the contraction of the Solar Nebula in order to understand the formation of the Solar System and to derive Planetary Distance Law. He took the view that the Solar Nebula contracted in steps of Roche Limit. Roche Limit is defined as the three dimensional distance on entering which the secondary body breaks into pieces due to tidal forces of the primary. Alternatively, it is the three-dimensional distance within which the primordial matter which is left behind around the primary, after its formation, does not get condensed into a secondary, due to tidal forces of the primary. In his paper entitled "Contraction of the Solar Nebula", Rawal (aforementioned papers) took the assumption that the ratio of the density of the primary (ρ_n) to the density of the secondary (ρ_s), which appears in the formula of Roche Limit, is of the order unity, that is, $(\rho_p/\rho_s) = 1$. In order to get closer look in the contraction of the Solar nebula, here, in this paper, we would like to remove this restriction on the ratio (ρ_n/ρ_s) and take it to be 0.7, 0.8, 0.9 or 1.1, 1.2, 1.3

and derive the distances of outer and inner edges of the gaseous rings, which one by one, go to form secondaries around the primary (here, the Sun), out of which planets were formed. This may give us closer look of the contraction of the Solar Nebula which is going to form the Solar System, giving rise to the form of Planetary Distance Law, consistent with 2/3-stable Laplacian Resonance Relation, which may be closer to reality. After going through this exercise, it is found, here, that the assumption that (ρ_p/ρ_s) = 1 may be relaxed. If it is less than 1, the system is shrunk and if it is more than 1 the system expands, only the Scale-parameter changes, the structure remains similar. However, in all these cases resonance necessarily will not be stable 2/3-Laplacian resonance. For stable 2/3-Laplacian resonant orbits, the ratio $(\rho_p/\rho_s) = 1$ is utmost necessary. One, therefore, concludes that the orbits in the Solar System are stable because the ratio (ρ_p/ρ_s) involved in the Roche Limit, is of the order unity.

Keywords: Contracting solar nebula; closer look.

1. INTRODUCTION

Since the time Copernicus (1542) pronounced that the planets revolve around the Sun, astronomers have been trying to understand the formation of the Solar System (Figs. 1, 2). Numerous theories for the formation of the Solar System have, so far, been advanced [1,3].

Fig. 1. The solar system

Fig. 2. The solar system

Among all these theories of the formation of the Solar System, Laplace's nebular hypothesis is favoured, as it explains (1) the isotopic abundance of the elements in the Solar System and in the Universe, (2) the estimates of the ages of the Sun and the planets, (3) Our understanding of transformation of a hot magnetic and rotating interstellar gas cloud into a star, (4) chemical and mineralogical composition of different objects in the Solar System such as meteorites, (5) Coplanarity of the orbital radii of almost all planets in the Solar System, (6) rotational directions of all planets, except Venus, are the same as the direction of rotation of the Sun, (7) directions of revolutions of all planets are the same as rotational direction of the Sun indicating the co-genetic origin and (8) support from astronomical observations [12]. However, it faced the following problems: (1) the problem of explaining extraordinary character of the distribution of mass and angular momentum in the Solar System, (2) the problem of explaining why the protosun shaded a discrete system of gaseous rings in which planets were formed and (3) the problem of explaining how the planets were aggregated from each gaseous ring [1 to 11].

The difficulties faced by Laplacian hypothesis were considered [13,14,15] and [13,14,15]. They presented an outline of the Laplacian theory, which they call modern Laplacian theory for the formation of the Solar System. They considered the influence of a supersonic turbulent convection and the radial turbulent stress in the cloud and showed that how this radial turbulent stress led to the formation and detachment of a discrete system of gaseous rings when the inward gravitational force balanced the outward centrifugal force, the ratio of the orbital radii R_p/R_{p-1} of successively disposed gaseous rings being a constant, forming a geometrical progression (Fig. 3), [12,13,14,15,16,17,18] and [1 to 11].

However, the ratio of the orbital radii R_p of successively disposed gaseous rings that they obtained is, of course, a constant forming a geometrical progression, but it does not lead to correct distances of planets after two or three steps, as the geometrical progression proceeds outward. Also, it is not found consistent with Kepler's third law of Planetary motion, and also with Laplace's stable 2/3 – resonance relation. It cannot generate Planetary Distance Law and even cannot explain the existence of rings around the central body.

1.1 Roche Limit

Roche Limit (Fig. 4) is defined as the three dimensional distance around the Primary on entering which the secondary body breaks into pieces due to tidal forces of the primary. Alternatively, it is the three-dimensional distance within which the primordial matter which is left behind around the primary, after its formation, does not get condensed into a secondary, due to tidal forces of the primary. Roche Limit [4] is given by

$$
d_{\text{Roche}} = 1.442 \left[\rho_p / \rho_s \right]^{1/3} x R \tag{1.1}
$$

where, R is the radius of the primary (the central body, here, the Sun), ρ_p is the density of primary and ρ_s is the density of secondary.

Rawal (1984) assumed $(\rho_p/\rho_s) = 1$ so that

$$
d_{Roche} = 1.442 R \tag{1.2}
$$

If we denote the Roche constant 1.44 which appears in Roche Limit formula as 'a' for convenience, we write

$$
d_{Roche} = a R \tag{1.3}
$$

Trying to understand the formation of the Solar System, [4] took the view that the solar nebula had contracted by steps of Roche Limit to form the Solar System. In this process, it was also seen that the phenomenon of supersonic turbulent convection and the radial turbulent stress described in [13,14,15,16] were operative and when the inward gravitational force became equal to the outward centrifugal force, a ring of material was shaded out (disposed of) (Fig. 6). This brought about a momentary halt at various stages of contraction. His aim was an attempt to understand and explain, on the basis of concept of Roche Limit and the Laplacian stable 2/3 resonance Relation the discrete distribution of planets, that is, planetary distance law in the Solar System and try to understand the physics underlying the formation of planetary System and the physical meaning of constants involved in the Planetary Distance Law.

In his paper, entitled "Contraction of the Solar Nebula", [14] took the assumption that the ratio of the density of the primary (ρ_p) to the density of the secondary (ρ_s) which appears in the formula of Roche Limit is of the order unity, that is, $(\rho_p/\rho_s) = 1$, and derived the Planetary Distance

Law in the form $r_n = R_0 a^n$, n=1, 2, 3, ……k, where R_0 is the radius of the primary, here, the Sun, and 'a' is a pure constant, we call it as the Roche Constant, the value of which, in the Solar System, is 1.442 (See Table 1).

In order to get closer look at the contracting Solar Nebula, here, in this paper, we would like to remove the restriction on the ratio (ρ_p/ρ_s) which appears in the formula of Roche Limit to be of order unity and take it to be 0.7, 0.8, 0.9 or 1.1, 1.2, 1.3 and derive the distances of outer and inner edges of the gaseous rings which, one by one, go to form secondaries, around the primary (here, the Sun), out of which planets were formed.

Fig. 3. Supersonic turbulent convection

Fig. 4. Roche limit

2. CONTRACTING SOLAR NEBULA

It is assumed that there was a spinning spherical gas cloud of interstellar gas and dust with mass, M, slightly greater than M_{\odot} (M_{\odot} = Sun's Mass), and certain radius, denoted by R_p . Under the influence of its self-gravitation, the cloud began to contract and due to the principle of conservation of angular momentum, it began to spin faster and faster as the nebula went on to contracting. Solar nebula also went onto bulging at the equator as it went to rotate faster and faster. A stage was reached at which the outward centrifugal force became equal to the inward gravitational force at the equator, at which point the gravitational and rotational instability arose along with supersonic turbulent convection and the radial turbulent stress, consistent with stable 2/3-Laplacian resonance relation. Laplacian Resonance Relation is [10]: $n_1 - 3$ $n_2 + 2n_3 = 0$ where n_1 , n_2 , n_3 are mean motions of triads of successive planets around the Sun. This resulted into a shell of gaseous matter which evolved into a ring of gaseous matter at the equator and got detached. This whole process got repeated itself, till the solar nebula reached its present size, in which form, we call it, the Sun. Here, the contraction of the above solar nebula halting at various radii is described in a particular fashion given below (Fig. 6). In this process, it is seen that the halts at various radii are being brought about by the phenomena of supersonic turbulent convection and the radial turbulent stress, when the outward centrifugal force became equal to the inward gravitational force, consistent with stable 2/3-Laplacian resonance relation. The supersonic turbulent convection does the following jobs: (1) It creates an additional source of outward pressure in a solar nebula called the radial turbulent stress which helps halt momentarily the free collapse of the Solar nebula by releasing a ring of matter, first at the dimensions of the Limit of the planetary system [6], (2) it causes the interior of the solar nebula to rotate almost uniformly like a rigid body due to a large turbulent viscosity and also drastically lowers the moment –of-inertia coefficient 'f' of the protosun, thereby, allowing the protosun to give up its angular momentum to a very light planetary system and (3) it leads to the formation of a very dense ring of gas and dust at the equator of the protosun, thereby, causing the protosun to dispose of its excess angular momentum through the successive detachment of a discrete system of rings of gas and dust.

Consider that the Solar nebula has initial radius R_p (Figs. 6 and 7). Assume that the Solar nebula has shrunk to a radius R_{p-1} for which R_p is the Roche Limit of the cloud with radius R_{p-1} (Figs. 6, 7, 8).

Fig. 5. Depending upon the mass of the nebula and its distance from the galactic centre, The Milky Way Galaxy assigns the Limit to the solar nebula out of which the Solar System was formed Gravitational collapse in the nebula starts from the limit of its span. This is the boundary to the Solar System

Fig. 6.

Schematic diagram showing the formation of the Solar System and the Satellite Systems of planets on the basis of contraction of the solar and sub-solar nebulae in terms of Roche Limit in the background of Modern Laplacian Theory.

Fig. 7.

Fig. 8.

Therefore, relation between R_p and R_{p-1} of the contracting spherical gas cloud (Solar nebula) can be written as

$$
R_p = a R_{p-1} \tag{2.1}
$$

The three-dimensional shell of gas and dust (matter) having width $R_p - R_{p-1}$ forms the Roche zone of the protosun having radius R_{p-1} . The matter in the shell having width R_{p} – R_{p-1} forms a bulge at the equator of the rotating gas cloud and settles down to from a ring of matter at the equator of width $R_p - R_{p-1}$. The matter inside such a ring grows to form planetesimals, but cannot form a full secondary (planet) there, because the matter inside such a ring is still inside the Roche Limit of the protosun having radius R_{p-1} . The matter inside such a ring has to wait for further contraction of the solar nebula to take place, so that it comes out of Roche Limit of the spherical cloud of gas and dust to form a full secondary (planet) there.

At the next stage of contraction of the Solar nebula, the cloud shrinks to a radius R_{p-2} such that

$$
R_{p-1} = a R_{p-2}
$$
 (2.2)

Therefore, eqn. (2.1) takes the form

$$
R_p = a. (a R_{p-2}) = a^2 R_{p-2}
$$
 (2.3)

The annular ring (R_{p-2}, R_{p-1}) of width $(R_{p-1} - R_{p-2})$ lies inside the Roche Limit of the protosun then having radius R_{p-2} . At this stage, the previous ring (R_{p-1}, R_p) of matter comes out of the Roche zone of the protosun having radius R_{p-2} , and the matter inside it, grows to form a planet, as it has come out of the Roche Limit.

Rawal (1984) assumed that the contraction of the Solar nebula proceeded in this fashion till the solar nebula reached its present size, in which form, we call it, the Sun, the halts at various radii were being brought about by the phenomena of supersonic turbulent convection and the radial turbulent stress, at the point at which the outward centrifugal force at the equator became equal to inward gravitational force, consistent with stable Laplacian 2/3-resonance relation, eventually leading us to the stage.

$$
R_1 = a R_{\odot} \tag{2.4}
$$

Where R_{\odot} is the radius of the present Sun. In terms of the radius of the present sun, the sequence of the radii of the contracting Solar nebula at various stages of the contraction can be expressed as

$$
R_p = R_{\odot} \ a^p \ , \qquad p = 1, 2, 3, 4, \dots, k \qquad (2.5)
$$

Table 1, shows various R_p . The known planet residing in the ring labelled (R_{p-1}, R_p) for various values of p are also mentioned.

On this scheme, eqn. (2.5) is looked upon as giving rise to outer and inner boundaries of various rings. The scenario, here, brings out that the ring structure feature is common and natural feature of the heavenly bodies, in particular, of the major members of the Solar System such as Jupiter, Saturn, Uranus, Neptune, our sun and even our own Milky Way Galaxy (Figs. 8, 9, 10, 11, 12, 13, 14, 15). It is obvious that the distant planets were formed earlier.

Fig. 9. Rings around the Planets

Fig. 10.

Fig. 11. Rings around the galaxy

Fig. 12. Rings around the Galaxy

Fig. 13. Rings around the Galaxy

Fig. 14. Rings around the Galaxy

Fig. 15. Rings around the Sun

Table 1.

Table 1. Continued…

On the basis of supersonic turbulent convection and the radial turbulent stress and the law of conservation of mass and angular momentum, [13] in the modern Laplacian theory of the Solar System, gets the ratio of the orbital radii R_p of successively disposed gaseous rings to be a constant given by:

$$
R_p/R_{p-1} = \left[1 + \frac{m}{M_f}\right]^2 = \text{Constant} \tag{2.6}
$$

Where m is the mass of the disposed ring, M, the remaining mass of the protosolar nebula and f, the moment –of-inertial coefficient. [13] distributed the solar material 0.05 M_{\odot} , (M_{\odot} = Solar mass) which has gone to form the planetary system [11,19,20,21,22] among twenty orbits that he got between the present Sun and Neptune, ten between Mercury and the present size of the Sun, and ten between Mercury and Neptune by putting $m = 1000 M_{\oplus}$ and $f = 0.01$ in the eqn (2.6) and got Bodes' constant to be 1.69 which is higher compared to Rawal's which is 1.442. As Rawal is getting twenty five rings between the present Sun and Neptune, each ring in his work gets mass 660 M_{\oplus} , as its share. Hence putting m =660 M_{\oplus} and f = 0.01, Rawal found

$$
R_p/R_{p-1} = \left[1 + \frac{m}{M_f}\right]^2 = \text{Constant} = 1.442 = a \tag{2.7}
$$

This is the difference between Rawal's work and that of Prentice. As geometric series progresses (goes ahead), the planetary distances deviate very much from their real values in Prentice's work, whereas in Rawal's work, the planetary distances remain closer to real values.

Without going into complex details of the theory of supersonic turbulent convection and turbulent stress, when the outward centrifugal force balances the inward gravitational force, that is, considering the rotational evolution of the protosun towards the point of rotational instability, one arrives at the relation Rawal (1986).

$$
R_p(\theta)/R_{p-1}(\theta) = \left[1 + \frac{\theta}{2}\right] \tag{2.8}
$$

Where θ = Inward gravitational force / outward centripetal force.

Thus, one inclines to conclude that during the time, the rotation parameter θ increases from O and attains the value 1, corresponding to the instability limit, the protosolar nebula decreases its radius from R_p to R_{p-1} , where R_p is the Roche Limit of the Solar nebula, then having radius R_{n-1} . At this time, supersonic turbulent convection dies down resulting in the steep density inversion at the equator and shadding out a ring of matter of width ($R_p - R_{p-1}$). At the rotational instability $\theta \rightarrow$ 1 and

 $R_p/R_{p-1} \rightarrow 3/2 = 1.5$ (2.9)

Just before instability arises

 θ = 0.9 and, we have

$$
R_p/R_{p-1} = \left(1 + \frac{0.9}{2}\right) = 1 + .45 = 1.45\tag{2.10}
$$

If θ = 0.89

$$
R_p/R_{p-1} = \left(1 + \frac{0.89}{2}\right) = 1 + 0.44 = 1.44\tag{2.11}
$$

This shows superiority of Rawal's work in this case

Several authors [13,15,17,18,1] have arrived at different forms of Titius-Bode Relation in their attempts to explain planetary distances. All were empirical relations. In comparison with those relations, eqn. (2.5) giving outer and inner boundaries of various rings of gas and dust has a physical interpretation, in the sense that it is based on the concept of Roche Limit applied to contracting solar nebula, the halts at various radii are being brought out by the phenomenon of supersonic turbulent convection and the radial turbulent stress, when the outward centrifugal force balances the inward gravitational force, that is, at rotational instability limit, consistent with stable 2/3-Laplacian resonance relation, leading to the formation and detachment of discrete system of rings of gas and dust. This provides an understanding and relation among supersonic turbulent convection and the radial turbulent stress, Roche Limit, rotational instability, stable Laplacian-2/3 resonance, in that rotational instability at the equator of spinning Proto Solar nebula arises at various stages of its quasi-static contraction precisely by the step of Roche Limit (Roche's Constant) consistent with supersonic turbulent convection and the radial turbulent stress and stable Laplacian-2/3 resonance, leading to the formation and detachment of a discrete system of rings of gas and dust, the whole process being controlled by the phenomenon of supersonic turbulent convection and the radial turbulent stress.

The usefulness and novel point of this work is that once the radius of the primary is known, the relation can be set up very simply and uniquely. Interestingly enough, it generates Planetary Distance Law, trying to understand physics behind it and trying to understand physical meaning of constants involved, physics behind the formation of the Solar System. Important

point about this is that if you know the diameter of the central celestial body (a star, galaxy, planet etc.), you know the structure of the whole system. In this connectipon one should go through [23,24,25].

3. CLOSER LOOK AT THE CONTRACTING SOLAR NEBULA

While theorizing and understanding the contraction of the Solar nebula, through the steps of Roche Limit, to understand the formation of the Solar System [4] assumed the ratio (ρ_p/ρ_s) involved in Roche Limit to be of order unity. Rawal felt that to be closer to reality, one should take closer look at the contracting solar nebula by taking this ratio to be 0.7, 0.8, 0.9, 1.1, 1.2, 1.3 and retrace out the Table 1. Calculations show that the assumption $(\rho_p/\rho_s) = 1$ may be relaxed, if it is less than 1, the system is narrowed down, and if it is more than 1, the system expands, only the scale-parameter changes, the structure remains similar. However, in all these cases (ρ_p/ρ_s) \neq 1 resonance necessarily will not be stable 2/3-Laplacian resonance. For stable 2/3-Laplacian resonant orbits, the ratios (ρ_p/ρ_s) = 1, is utmost necessary. One, therefore, concludes that the orbits in the solar system are stable because the ratio (ρ_p/ρ_s) involved in Roche Limit is of the order unity.

4. CONCLUSION

The closer look at the contracting Solar nebula shows that it is not necessary to take the ratio (ρ_p/ρ_s) involved in the formula for Roche Limit to be unity. This condition may be relaxed. If it is less than 1, the system is narrowed down, and if it is more than 1, the system expands, only the scale parameter changes, the structure remains similar. However, in all these cases $(\rho_p/\rho_s) \neq 1$ resonance necessarily will not be stable 2/3- Laplacian resonance. For stable 2/3-Laplacian resonant orbits, the ratios (ρ_n/ρ_s) = 1, is utmost necessary. One, therefore, concludes that the orbits in the solar system are stable because the ratio (ρ_n/ρ_s) involved in Roche Limit is of the order unity.

ACKNOWLEDGEMENTS

Author thanks Shri Narhari Koya, an Industrialist, from Kolkata for his financial help for carrying our research work at the Indian Planetary Society. Thanks are due to all five reviewers for their critical comments and useful suggestions which helped improve the content of the Paper.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

REFERENCES

- 1. Rawal JJ. Bull. Astr. Soc. India. 1978;6: 92-95.
- 2. Rawal JJ. Is there a ring around milky way? Indian Journal of Radio and Space Physics. 1982;11:100-101.
- 3. Rawal JJ. Contraction of the solar nebula. Earth, Moon and Planets. 1984;31:175- 182.
- 4. Rawal JJ. Wasaburo Unno and Tomio Asai (ends.) "Astrophysical and Geophysical Convection" Proceedings and Mini Symposium held in the Department of Astronomy University of Tokyo. October 11-12, 1984;70-85.
- 5. Rawal JJ. Astrophysics and Space Science. 1986;119:159-166.
- 6. Rawal JJ. Bulletin, Astronomical Society India. 1986;14:15-19.
- 7. Rawal JJ. Earth, Moon and Planets. 1986; 34:93-100.
- 8. Rawal JJ. Earth, Moon and Planets. 1989; 44:265-274.
- 9. Rawal JJ. Journal of Astrophysics and Astronomy. 1989;10:257-259.
- 10. Rawal JJ. The Moon and the Planets. 1981;24:407-414.
- 11. Rawal JJ, Ramadurai S. Earth, Moon and Planets. 2012;108:95-99.
- 12. Reeves H. in the Origin of the Solar System, Dermott, S.F. (ed.); 1978.
- 13. Ter Haar D, Cameron AGW. In the Origin of the Solar System, R. jastrow and A.G.W. Cameron (Eds.), Academic Press, New York. 1963;1.
- 14. Prentice AJR. In 'The Origin of the Solar System', Dermott, S.F. (ed.), Wiley, London. 1978;111.
- 15. Ter Haar D. Ann. Rev. Astron. Astrophys. 1967;5:267.
- 16. Urey HC. Geochim, Cosmochim. Acta 1, 209.Viley, London. 1951;1.
- 17. Nieto MM. The Titius-Bode Law of Planetary Distances, Its History and Theory, Pergamon Press, Oxford; 1972.
- 18. Dermott SF. Monthly Notices Roy. Astron. Soc. 1968;141:363.
- 19. Kuiper GP. In J.A. Hynek (ed) Astrophysics, McGraw-Hill, New Yark, Chap.8; 1951.
- 20. Hoyle F. Quart. J. Roy. Astron.Soc. 1960; 1:28.
- 21. Hoyle F, Wickramasinghe NC. Condensation of the planets. Nature. 1968;217:415.
- 22. Andrew J. Hesselbrock and David A. Minton. The Astronomical Journal. 2019; 157:13.
- 23. Weidner C, Horne K. Astron. & Astroph. 521, Article No. A76; 2012.
- 24. Whipple FL. Earth, Moon and Planets, Harvard Univ. Press, Cambridge; 1971.
- 25. Zoe M, et al. Tidal disruption of satellites and formation of narrow rings. Mon. Not. R. Astron. Soc. 2012;424:1419-1431.

 $_$, and the set of th *© 2021 Rawal and Malhotra; This is an Open Access article distributed under the terms of the Creative Commons Attribution License (http://creativecommons.org/licenses/by/4.0), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.*

> *Peer-review history: The peer review history for this paper can be accessed here: https://www.sdiarticle4.com/review-history/70213*