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Study on Determining the Blood Glucose Concentration During an Intravenous Injection Using Volterra Integral Equations

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Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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Abstract

In this study, the mathematical model for determining the blood glucose concentration during an intravenous injection has been solved throw Volterra integral equation and using Chebyshev spectral method. The method is based on the blending of the Chebyshev pseudo spectral method and its implementation procedure reduces the Volterra integral equation to a system of algebraic equations that are easy to solve. For the practical

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application of the method, a mathematical model in medical science for determining the blood glucose concentration during an injection has been solved. Tables and figures were generated to verify the accuracy and convergence of the method. The results demonstrate that the method is efficient, convergent and accurate to the exact solution.

Keywords: Numerical methods; Chebyshev collocation method; Volterra integral equations.

1 Introduction

Many applications of engineering, biology, applied science and medicine can be expressed mathematically and solved by using single or system Volterra integral equations. In general, finding analytic solutions of Volterra Integral equations are usually difficult so it is required to obtain an approximate solution. Therefore, Volterra Integral equations have been of great interest by several authors and scientists. There are many analyticall numerical methods have been introduced, discussed and modified for finding a solution for linear Volterra integral equation, such as, Galerkin methods, Collocation methods, Taylor expansion, converting equation to a system of algebraic equations, Legendre wavelets method, Taylor polynomials and Power series method and expansion method [1–9]. Recently, Chauhan and Aggarwal [10] used Laplace transform for solving linear Volterra integral equation of second kind, Aggarwal et. all [11] applied Shehu transform for handling Volterra integral equations of first kind, Barycentric - Maclaurin interpolation method has been applied by [12] for solving Volterra integral equations of the second kind, Khidir [13, 14] suggest a highly accurate technique for solving Volterra integral equations based on the blending of the Chebyshev pseudo methods. Chebyshev spectral collocation methods have been applied successfully in different fields of sciences because of their ability to give very high accurate solutions of single or system of boundary value problems, this is because Chebyshev spectral methods are defined everywhere in the computational domain [15–24]. Therefore, it is easy to compute a high accurate values of a considered function at any point of the domain.

For some problems, it is impractical to use analytical methods because their solution process becomes too cumbersome, and convergence to the true solution can be very slow or not possible at all. For, this reason numerical methods are by far the most practical way of seeking solutions to the highly nonlinear systems. Solutions for some integral equations using analytical methods are difficult to found, so the quest for the most optimal method of solving problems is what drives, ever growing interest in the development of new methods and the modification and improvement of existing analytical and numerical methods. The prime objective of this paper is to present a new numerical method of solving integral equations that seeks to address some of the aforementioned numerical difficulties. We propose a very simple, yet very accurate and convergent iterative algorithm for solving linear integral equations.

In this work, we apply a new technique for solving linear Volterra integral equations that uses Chebyshev spectral collocation method. The implementation of the method that convert the integral equation into a system of linear algebraic equations by using the proposed operational integral matrix of known entries instead of the integration operator.

The main advantages of this method are that (i) this technique suggests a standard way of choosing the linear operator of the integral equation whereas the other related methods are choosing a linear operator to be simple in order to ensure that the integral equation can be easily solved and (ii) this algorithm transforms the integral equation into a system of linear algebraic equations that easier and faster to solve when compared to a system of integral equations.

The applicability, accuracy, and reliability of the method are confirmed by applying the method on the field of medical science for determining the glucose concentration in blood of a patient.

The paper is organized as follow: in section 2, we introduce a description of the proposed method. In section 3, we applied the method for solving linear Volterra integral equation. The numerical results are discussed and investigated in section 4. Finally, the paper is concluded in section 5.

2 Description of the Method

consider the following linear Volterra integral equation of the second kind given by

$$u(x) + \lambda \int_0^x K(x,t)u(t)dt = f(x), \quad 0 \leq x, t \leq b, \tag{1}$$

where unknown function $u(x)$ is the solution to be determined, the kernel $K(x,t)$ and the function $f(x)$ are given real valued functions,λ is a parameter. It is to be noted here that both the kernel $K(x,t)$ and the function $f(x)$ are known functions.

To illustrate the idea of the algorithm, we assume that the kernel $K(x,t)$ can be expressed as a product of two functions namely $w(x)$ and $v(t)$, consequently. Equation (1) can be obtained as follows:

$$u(x) + w(x) \int_0^x v(t)u(t)dt = f(x). \tag{2}$$

Now, let us expressed the integral term of the integral equation above as

$$\int_0^x v(t)u(t)dt = \Phi(x). \tag{3}$$

One approach is to note that the integral equation (3) is an initial value problem obtained as

$$\frac{d\Phi}{dx} = v(x)u(x), \text{ with } \Phi(0) = 0. \tag{4}$$

This differential equation is very simple and can be solved using any method, her we used the Chebyshev spectral collocation method, the functions $\Phi(x), v(x)$ and $u(x)$ are approximateed as a truncated series of Chebyshev polynomials given by the form [25–27]

$$\Phi(x) \simeq \Phi(x_j) = \sum_k^N \tilde{\Phi}_k T_k(x_j),$$
$$u(x) \simeq u(x_j) = \sum_k^N \tilde{u}_k T_k(x_j),$$
$$v(x) \simeq v(x_j) = \sum_k^N \tilde{v}_k T_k(x_j),$$

where T_k is the kth Chebyshev polynomial and $, \tilde{\Phi}, \tilde{u}, \tilde{v}$ are the Chebyshev coefficients and x_j are the Gauss-Lobatto collocation points [27] defined by

$$x_j = \frac{1}{2} x_N (1 - \cos \frac{j\pi}{N}), \qquad j = 0, 1, 2, ..., N, \tag{5}$$

where $N + 1$ is the number of collocation points (the nodes). The derivative of Φ at the collocation points is represented as

$$\frac{d\Phi}{dx} \simeq \sum_{k=0}^N \mathcal{D}_{jk} \Phi = \mathcal{D}\Phi, \tag{6}$$

where $\mathcal{D}$ the Chebyshev spectral differentiation matrix whose entries are given by [25]

$$\left.\begin{array}{ll} \mathcal{D}_{00} = \frac{2N^2+1}{6} & , \\ \mathcal{D}_{jk} = \frac{cj}{ck}\frac{(-1)^{j+k}}{x_j - x_k}, & j \neq k, j, k = 0, 1, 2, ..., N, \\ \mathcal{D}_{kk} = \frac{x_k}{1-x_k^2}, & k = 1, 2, ..., N-1, \\ \mathcal{D}_{NN} = -\frac{2N^2+1}{6} & \end{array}\right\}, \tag{7}$$

here $c_0 = c_N = 2$ and $c_j = 1$ with $1 \leq j \leq N-1$.

Substituting the above assumptions in equation (4) yields a system of algebraic equations expressed as the following matrix equation

$$\begin{bmatrix} \mathcal{D}_{0,0} & ... & \mathcal{D}_{0,N} \\ \mathcal{D}_{1,0} & ... & \mathcal{D}_{1,N} \\ \vdots & & \vdots \\ \mathcal{D}_{N,0} & ... & \mathcal{D}_{N,N} \end{bmatrix} \begin{bmatrix} \Phi_0 \\ \Phi_1 \\ \vdots \\ \Phi_N \end{bmatrix} = \begin{bmatrix} v_0 \\ v_1 \\ \vdots \\ v_N \end{bmatrix} \begin{bmatrix} u_0 \\ u_1 \\ \vdots \\ u_N \end{bmatrix}, \tag{8}$$

where $\Phi_i = \Phi(x_i), u_i = u(x_i)$ and $v_i = v(x_i)$. The solution of this system for $\Phi(x_i)$ is obtained by

$$\begin{bmatrix} \Phi_0 \\ \Phi_1 \\ \vdots \\ \Phi_N \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & ... & 0 \\ \mathcal{D}_{1,0} & & & & \mathcal{D}_{1,N} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathcal{D}_{N,0} & & & & \mathcal{D}_{N,N} \end{bmatrix}^{-1} \begin{bmatrix} v_0 \\ v_1 \\ \vdots \\ v_N \end{bmatrix} \begin{bmatrix} u_0 \\ u_1 \\ \vdots \\ u_N \end{bmatrix} - \begin{bmatrix} u_0 v_0 \\ u_0 v_0 \\ \vdots \\ u_0 v_0 \end{bmatrix}. \tag{9}$$

Here we observe that the first row of Chebyshev differential matrix $\mathcal{D}$ in equation above is replaced by the row [1, 0, 0, ...] and we subtracted the vector $[u_0 v_0, u_0 v_0, ..., u_0 v_0]^T$, this is caused by imposing the condition $\Phi(0) = 0$ into the system of linear equation (8). According to the integral equation (3) and equation (9), we introduce the following integral operator $\mathbf{L}_{[v]}(u)$ defined as

$$\mathbf{L}_{[v(x)]} u(x) = u(x_0)v(x_0) + \int_0^x v(x)u(x)dx, \tag{10}$$

where $u = [u_0 \;\; u_1 \;\; ... \;\; u_N]^T, [v] = [\, v_0 \;\; v_1 \;\; ... \;\; v_N]^T$ and

$$\mathbf{L} = \begin{bmatrix} 1 & 0 & ... & 0 \\ \mathcal{D}_{1,0} & ... & & \mathcal{D}_{1,N} \\ \vdots & & & \vdots \\ \mathcal{D}_{N,0} & ... & & \mathcal{D}_{N,N} \end{bmatrix}^{-1}. \tag{11}$$

The integral operator $\mathbf{L}$ is a square matrix of size $(N+1) \times (N+1)$.

2.1 The linearity of the operator L

It is clear that the operator defined by equation (10) is a linear operator since it is a an integral operator,

$$\int_0^x \sum_{i=1}^n V_i(x)V_i(x)dx = \sum_{i=1}^n \mathbf{L}_{[V_i(x)]} U_i(x) - \sum_{i=1}^n U_i(x_0)V_i(x_0) \tag{12}$$

2.2 Existence and uniqueness of the operator L

In this sub-section, we show that the operator $\mathbf{L}$ is exist and unique.

Theorem
(i) Any square matrix A is invertible (nonsingular) if and only if the determinant of (A) is non zero.
(ii) If A is an invertible matrix, then its inverse is exist and unique.
Proof:
According to the theorem above, the existence and uniqueness of the linear operator $\mathbf{L}$ depends on the existence and uniqueness of inverse of matrix

$$\begin{bmatrix} 1 & 0 & ... & 0 \\ \mathcal{D}_{1,0} & ... & & \mathcal{D}_{1,N} \\ \vdots & & & \vdots \\ \mathcal{D}_{N,0} & ... & & \mathcal{D}_{N,N} \end{bmatrix}$$

The determinants of matrix above have been computed on the domain $x \in [0, b]$ for $N = 5, 6, 7, ..., 100$. In Fig. 1, we plotted the computed determinants for different values of b varied N. We observe that from the Figure all the determinants are not equal zero and it is notice that all the determinants is greater than or equal 1 i.e. the inverse of the operator $\mathbf{L}$ is exist and unique.

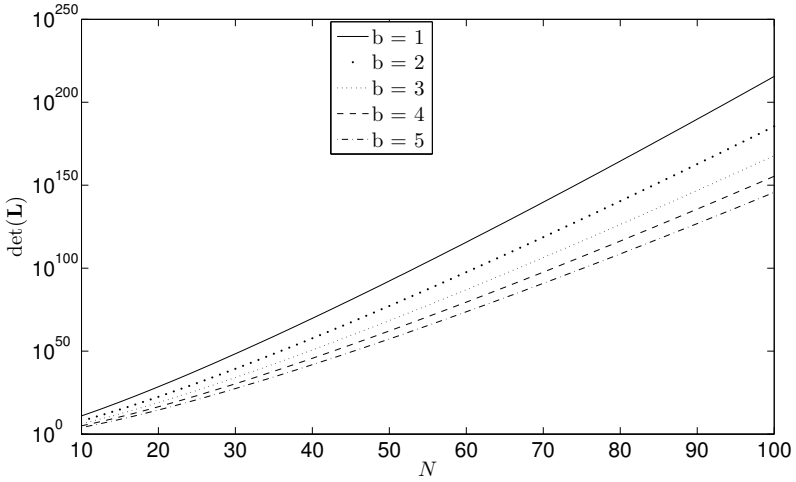

Fig. 1. The determinatns (det) of the matrices L.

The general idea underpinning the use of the proposed method is to convert the linear Volterra integral equation into a system of linear algebraic equations that replace the integral parts in the integral equation by the integral matrices operators. The obtained linear algebraic equations can easily be solved with the help of symbolic computation software such as Maple, Mathematica, MATLAB, or other symbolic computer packages.

3 Application

For a physical explanation of the present scheme, we consider a problem from the field of medical science for determining blood glucose concentration of a patient at any time. Mathematically, this model can be written in terms of linear Volterra integral equation as [28–31]

$$C(t) + k \int_0^t C(x)dx - \frac{\alpha}{V} t = C_i, \quad \text{with } C(0) = C_i \tag{13}$$

where:
$C(t)$ is the blood glucose concentration at time t,
k is the constant velocity of elimination (in 1/min).

α is the proportion of the of infusion (in mg/min).
V is the volume in which glucose is distributed (in dL).
C_i is the initial concentration of glucose in the blood (in mg/dL).

Applying the proposed algorithm on equation (13) and according to the assumptions (6) and (10) and using the integral matrix $\mathbf{L}$ defined by equation (10), one can transformed the integral equation (13) into a system of linear algebraic equations given by

$$\overrightarrow{\mathbf{C}} + k\overrightarrow{\mathbf{L}}\overrightarrow{\mathbf{C}} = C_i + \frac{\alpha}{V}\overrightarrow{\mathbf{t}} + kC_i, \tag{14}$$

$$\left[\overrightarrow{\mathbf{I}} + k\overrightarrow{\mathbf{L}}\right]\overrightarrow{\mathbf{C}} = C_i + \frac{\alpha}{V}\overrightarrow{\mathbf{t}} + kC_i, \tag{15}$$

$$\overrightarrow{\mathbf{A}}\overrightarrow{\mathbf{C}} = \overrightarrow{\mathbf{f}}. \tag{16}$$

where:
$\overrightarrow{\mathbf{C}}$: is unknown vector column of size $(N+1)\times 1$,
$\overrightarrow{\mathbf{I}}$: is an identity matrix of size $(N+1)\times(N+1)$,
$\overrightarrow{\mathbf{t}}$: is known column vector of size $(N+1)\times 1$ and defined by $\overrightarrow{\mathbf{t}} = [t_0, t_1, ..., t_N]^T$, T stands to transpose,
$\overrightarrow{\mathbf{f}}$: is known column vector of size $(N+1)\times(N+1)$ and defined by $\overrightarrow{\mathbf{f}} = C_i + \frac{\alpha}{V}\overrightarrow{\mathbf{t}} + kC_i$.
$\mathbf{A} = \overrightarrow{\mathbf{I}} + k\overrightarrow{\mathbf{L}}$.

Thus, the final solution of the Volterra integral equation (13) is obtained as

$$\overrightarrow{\mathbf{C}} = \overrightarrow{\mathbf{A}^{-1}}\overrightarrow{\mathbf{f}}. \tag{17}$$

The solution given by equation (17) gives the values of blood glucose concentration $C(t)$ at time t for various values of glucose concentration at initial state C_i, the rate of infusion α and constant velocity of elimination k and the volume V in which glucose is distributed,.

4 Results and Discussion

In this section we present and analyze the obtained results of determine the blood glucose concentration which is modeled by solving the linear Volterra integral equation. Implementation of the numerical schemes was performed using personal computer of 2.5 GHz CPU speed including Matlab software to perform the simulation results. The values of blood glucose concentration are computed plotted at different values time, constant velocity of elimination, the proportion of the of infusion, the volume in which glucose is distributed, and initial concentration of glucose in the blood. The accuracy of the method is demonstrated by presenting infinity error norms $C_E(t)$ between exact and approximate results. The computational times for all obtained results have been generated to confirm the speed and computational efficiency of the current technique. All the results are showed in Tables 1-5 and Figs. 2-6.

Table 1 shows the maximum absolute errors for $C(t)$ between the present and exact solutions at selected values of the parameters C_i, α, V and k. Also, the times taken for the computation have been presented. A striking feature of the proposed method is that a high level of accuracy is achieved and also the method gives very small errors without using any iterations to decrease the error as applied in most of the iterative methods. Also, the present algorithm is computationally fast as accurate results are generated in a fraction of a second as it shown in Table 1.

In Fig. 2. we display a comparison between the exact and numerical results of $C(t)$ at various values of the parameters C_i, α, V and k. The figures indicate that there is very good match between the two sets of results even at no iteration of the current technique compared with the exact results.

Table 1. Comparison between maximum absolute errors of $C(t)$ with various choices of C_i, α, V and k.

Paramenter		$C(t)$	CPU time (sec)
C_i	320	$4.647e-012$	0.006
	324	$4.803e-012$	0.008
	328	$4.675e-012$	0.009
α	280	$4.647e-012$	0.013
	282	$4.647e-012$	0.015
	284	$4.832e-012$	0.017
V	45	$4.647e-012$	0.023
	47	$4.576e-012$	0.025
	49	$4.420e-012$	0.026
k	0.058	$4.420e-012$	0.029
	0.060	$4.349e-012$	0.030
	0.062	$4.292e-012$	0.031

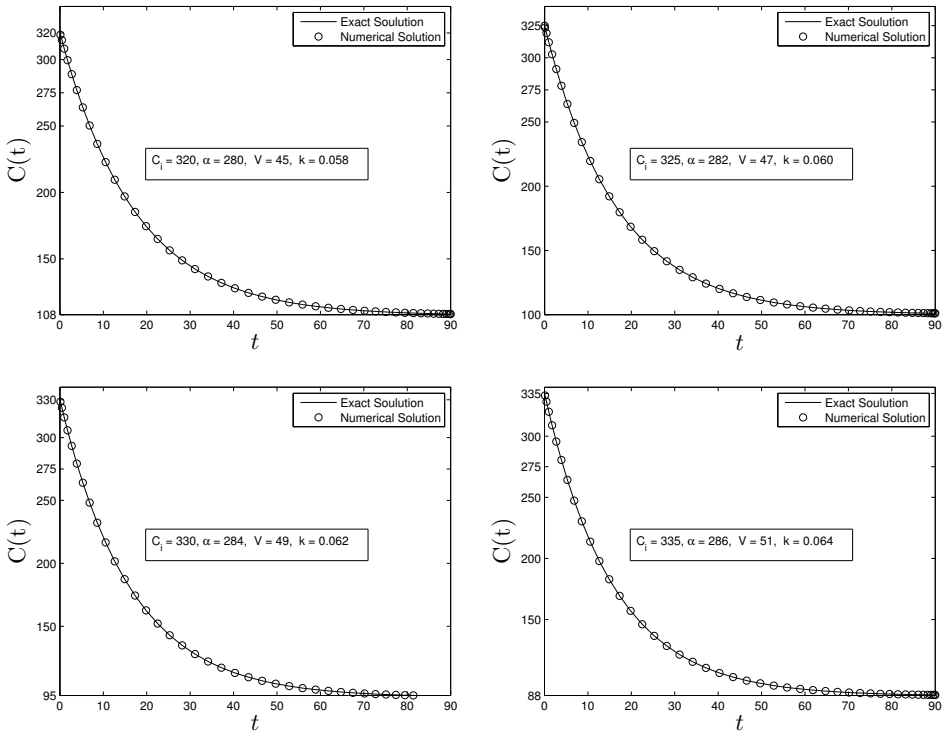

Fig. 2. Comparison between the numerical and exact solutions.

In Table 2, we presented the results of blood glucose concentration $C(t)$ for various values of C_i and time t with fixed values of α, V and k.

From Table 2, it can be observed that, as time increases, blood glucose concentration decreases for all selected values of initial concentration C_i.

It is also notice that the time for achieving normal blood glucose concentration $C(t)$ increases as the initial concentration of glucose C_i decreases, this results is supported by the graph, which is plotted in Fig. 3.

Table 2. Blood glucose concentration $C(t)$ for various values of C_i and time t with $\alpha = 280, V = 45$ and $k = 0.058$.

Time	initial concentration of glucose C_i				
t	320	325	330	335	340
0	320.00	325.00	330.00	335.00	340.00
10	226.38	229.18	231.98	234.78	237.58
20	173.96	175.53	177.10	178.67	180.23
30	144.62	145.49	146.37	147.25	148.13
40	128.18	128.68	129.17	129.66	130.15
50	118.98	119.26	119.53	119.81	120.08
60	113.83	113.99	114.14	114.30	114.45
70	110.95	111.04	111.12	111.21	111.29
80	109.33	109.38	109.43	109.48	109.53
90	108.43	108.46	108.48	108.51	108.54
CPU time	0.675/sec				

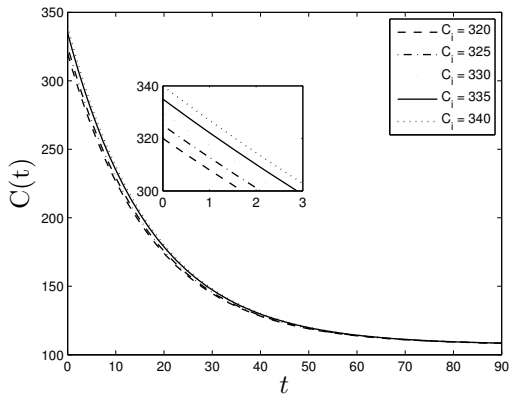

Fig. 3. Blood glucose concentration $C(t)$ for various values of $C_i$and time t with $\alpha = 280, V = 45$ and $k = 0.058$.

The result of blood glucose concentration $C(t)$ is showed in Table 3 for various values of rate of infusion (α) and time t when the parameters C_i, V and k are consider as fixed at $280, 45$ and 0.058, respectively. It is clear that,

as time t increases, blood glucose concentration $C(t)$ decreases for all selected values of rate of infusion (α). Fig. 4 confirms and supports the results obtained by Table 3.

Table 3. Blood glucose concentration $C(t)$ for various values of α and time t with $C_i = 280, V = 45$ and $k = 0.058$.

Time	rate of infusion α				
t	280	281	282	283	284
0	320.00	325.00	330.00	335.00	340.00
10	226.38	229.18	231.98	234.78	237.58
20	173.96	175.53	177.10	178.67	180.23
30	144.62	145.49	146.37	147.25	148.13
40	128.18	128.68	129.17	129.66	130.15
50	118.98	119.26	119.53	119.81	120.08
60	113.83	113.99	114.14	114.30	114.45
70	110.95	111.04	111.12	111.21	111.29
80	109.33	109.38	109.43	109.48	109.53
90	108.43	108.46	108.48	108.51	108.54
CPU time	0.705/sec				

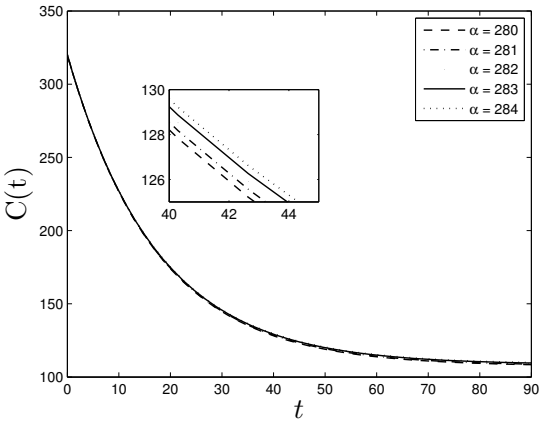

Fig. 4. Blood glucose concentration $C(t)$ for various values of α and time t with $C_i = 230, V = 45$ and $k = 0.058$.

In Table 4. presents the computed results of blood glucose concentration $C(t)$ for various values of the volume V and time t with fixed values of $C_i = 320, \alpha = 280$ and $k = 0.058$. From Table 4, it can be noted that, as time increases, blood glucose concentration decreases for all selected values of volume V. The same conclusion is confirmed by the graph 4.

Table 4. Blood glucose concentration $C(t)$ for various values V and time t with $C_i = 320, \alpha = 280$ and $k = 0.058$.

Time	volume V				
t	45	46	47	48	49
0	320.00	320.00	320.00	320.00	320.00
10	226.38	225.36	224.37	223.43	222.53
20	173.96	172.36	170.83	169.36	167.95
30	144.62	142.69	140.85	139.09	137.40
40	128.18	126.08	124.07	122.14	120.29
50	118.98	116.78	114.67	112.65	110.71
60	113.83	111.57	109.41	107.33	105.35
70	110.95	108.66	106.46	104.36	102.34
80	109.33	107.02	104.81	102.69	100.66
90	108.43	106.11	103.89	101.76	99.72
CPU time	0.686/sec				

Table 5 concluded that, as time increases from 0 to 90, blood glucose concentration $C(t)$ decreases at various values of velocity of elimination k when the other parameters C_i, α and V are fixed at $320, 280$ and 45, respectively. Fig. 6 plotted the blood glucose concentration $C(t)$ against time t for selected values of velocity of elimination k when $C_i = 320, \alpha = 280$ and $V = 45$. From Fig. 6, it is clear that as time t increases, the blood glucose concentration $C(t)$ decreases for all selected values of velocity of elimination k.

Table 5. Blood glucose concentration $C(t)$ for various values of k and time t with $C_i = 320, \alpha = 280$ and $V = 45$.

Time	velocity of elimination k				
t	0.058	0.060	0.062	0.064	0.066
0	320.00	320.00	320.00	320.00	320.00
10	226.38	222.41	218.51	214.69	210.94
20	173.96	168.85	163.92	159.16	154.57
30	144.62	139.46	134.55	129.88	125.44
40	128.18	123.33	118.75	114.44	110.38
50	118.98	114.47	110.25	106.30	102.60
60	113.83	109.61	105.68	102.01	98.58
70	110.95	106.95	103.22	99.75	96.50
80	109.33	105.48	101.90	98.55	95.43
90	108.43	104.68	101.19	97.92	94.87
CPU time	0.725/sec				

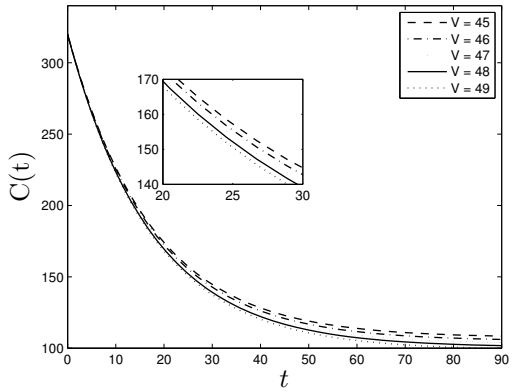

Fig. 5. Blood glucose concentration $C(t)$ for various values of V and time t with $C_i = 320, \alpha = 280$ and $k = 0.058$

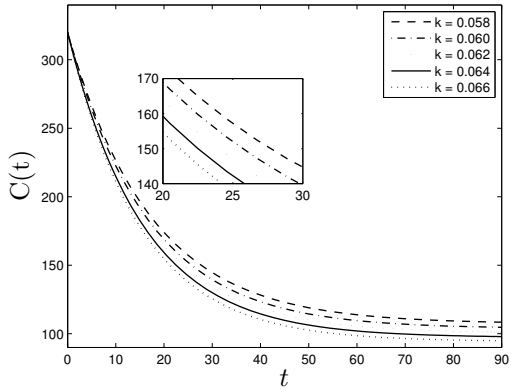

Fig. 6. Blood glucose concentration $C(t)$ for various values of k and time t with $C_i = 320, \alpha = 280$ and $V = 45$.

5 Conclusion

In this study, authors fruitfully applied new technique for solving linear Volterra integral equation. The technique suggested a new matrix used instead of the integral operator together with Chebyshev pseudospectral method. The using of this operational integral matrix allow us to replaced the integral equation to a system of linear algebraic equations. The efficient and reliability of the method are confirmed and applied on medical field during an intravenous injection for finding the concentration of blood glucose for patient at any particular time. This method gives the solution of problem of blood glucose concentration and provides us the required duration to achieve normal blood glucose concentration, which is very important for patients with diabetes mellitus. The numerical solutions have been shown in tables and graphs and also compared with the exact solutions. It found that the technique is very accurate and easy to apply and it is sufficient to give good agreement with the exact solution. Also, we can conclude that from this study, as time increases, blood glucose concentration decreases at all values of the constant velocity of elimination, the proportion of the of infusion, the volume in which glucose is distributed, and the initial concentration of glucose in the blood. Finally, the proposed method described above is useful method in solving Volterra and Fredholm integral equations and can be generalized for non linear Volterra-Fredholm integro-differential equations.

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Competing Interests

Authors have declared that no competing interests exist.

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