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Research Article

Classification of All Single Traveling Wave Solutions of Fractional Coupled Boussinesq Equations via the Complete Discrimination System Method

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In this paper, the complete discrimination system method is used to construct the exact traveling wave solutions for fractional coupled Boussinesq equations in the sense of conformable fractional derivatives. As a result, we get the exact traveling wave solutions of fractional coupled Boussinesq equations, which include rational function solutions, Jacobian elliptic function solutions, implicit solutions, hyperbolic function solutions, and trigonometric function solutions. Finally, the obtained solution is compared with the existing literature.

1. Introduction

The coupled system is composed of two or more differential equations (include ordinary differential equations, partial differential equations fractional partial differential equations, and stochastic partial differential equations) [1–3]. It is a very important class of mathematical and physical equations. In recent years, coupled systems have been widely studied by scholars because they come from physics, chemistry, communication, and engineering [4–8]. Among them, constructing the exact traveling wave solution of this kind of coupled system is a very important topic. Many meaningful methods have been proposed to solve the exact solutions of coupled systems, including Lie symmetry analysis [9], the method of dynamical systems [10, 11], Fan subequation method [12], generalized Jacobi elliptic function expansion method [13], extended modified auxiliary equation mapping method [14], and extended modified auxiliary equation mapping method [15].

The fractional coupled Boussinesq equations [16, 17] are a very important coupled system, which is usually used to simulate nonlinear shallow water surface wave phenomena.

$$\begin{cases} D_t^{\alpha} u(t,x) + D_x^{\beta} v(t,x) = 0, \\ D_t^{\alpha} v(t,x) + a D_x^{\beta}\left(u^2(t,x)\right) - b D_{xxx}^{3\beta} u(t,x) = 0, \end{cases} \quad 0 < \alpha, \beta \le 1, b \ne 0. \tag{1}$$

In [16], Muhannad et al. constructed the traveling wave solutions of fractional coupled Boussinesq equations by using the modified extended Tanh method. In [17], Khatun et al. obtained some explicit solutions of fractional coupled Boussinesq equations by the double $(G'/G, 1/G)$ expansion method. The work of references [16, 17] is based on the Jumarie's modified Riemann-Liouville derivative to study fractional coupled Boussinesq equations. Unfortunately, many literatures [18–20] have reported that the Jumarie's modified Riemann-Liouville derivative do not satisfy the chain rule and Leibniz formula. Therefore, it is urgent to find a new fractional derivative that can not only satisfy the chain rule but also obey Leibniz formula. In [21], Khalil et al. gave the definition and properties of the fractional derivative named conformable fractional derivative, which satisfy the above two conditions. The main purpose of this paper is to attain the exact traveling wave solutions of fractional coupled Boussinesq equations in the sense of conformable fractional derivative by using the complete discrimination system method [22–24].

Next, we review the definition of conformable fractional derivatives.

Definition 1. Let $f : [0,\infty) \longrightarrow R$. Then, the conformable fractional derivative of f of order α is defined as

$$D_t^\alpha f(t) = \lim_{\varepsilon \longrightarrow 0} \frac{f\left(t + \varepsilon t^{1-\alpha}\right) - f(t)}{\varepsilon}, \forall t \in [0,+\infty), \alpha \in (0,1]. \tag{2}$$

The function f is α-conformable differentiable at a point t if the limit in Equation (2) exists.

Theorem 2. *Assume that* $f, g : (0,\infty) \longrightarrow R$ *be differentiable and also α differentiable functions, then chain rule holds*

$$D_t^\alpha (f \circ g)(t) = t^{1-\alpha} g(t)^{\alpha-1} g'(t) D_t^\alpha (f(t))\big|_{t=g(t)}. \tag{3}$$

The structure of this paper is as follows. In Section 2, we simplify Equation (1) to nonlinear ordinary differential equations by fractional traveling wave transformation. Then, the complete discrimination system is used to construct the classification of all single traveling wave solutions of fractional coupled Boussinesq equations. In Section 3, we give a summary.

2. Exact Solutions of System (1)

Now, we introduce the transformation

$$\xi = \frac{x^\beta}{\beta} - c\frac{t^\alpha}{\alpha}, \tag{4}$$

where c is constant.

Substituting (4) into Equation (1) and integrating it with respect to ξ, we obtain

$$\begin{cases} c_1 - cu + v = 0, \\ c_2 - cv + au^2 - bu'' = 0, \end{cases} \tag{5}$$

where c_1 and c_2 are integral constants.

From the first equation of Equation (5), we have

$$v = cu - c_1. \tag{6}$$

Substituting (6) into the second equation of Equation (5), we obtain

$$c_2 + c_1 c - c^2 u + au^2 - bu'' = 0. \tag{7}$$

Multiplying Equation (8) by u' and integrating it with respect to ξ, we obtain

$$\left(u'\right)^2 = \frac{2a}{3b}u^3 - \frac{c^2}{b}u^2 + \frac{2c_2 + 2c_1 c}{b}u + 2c_3, \tag{8}$$

where c_3 is the integral constant.

Suppose $\phi = (2a/3b)^{1/3}u$, $b_2 = -(c^2/b)(2a/3b)^{-2/3}$, $b_1 = (2c_2 + 2c_1 c/b)(2a/3b)^{-1/3}$, and $b_0 = 2c_3$. Hence, Equation (8) can be changed to

$$\left(\phi'\right)^2 = \phi^3 + b_2\phi^2 + b_1\phi + b_0. \tag{9}$$

Assume that $f(\phi) = \phi^3 + b_2\phi^2 + b_1\phi + b_0$, $\Delta = -27(2b_2^3/27 + b_0 - b_0 b_1/3)^2 - 4(b_1 - b_2^2/3)^3$, and $D_1 = b_1 - b_2^2/3$. Then, Equation (9) can be written in the following integral form:

$$\pm\left(\frac{2a}{3b}\right)^{1/3}(\xi - \xi_0) = \int \frac{d\phi}{\sqrt{\phi^3 + b_2\phi^2 + b_1\phi + b_0}}, \tag{10}$$

where ξ_0 is the integration constant. Then, we will obtain the classification of all solutions of Equation (10).

Case 1. If $\Delta = 0$ and $D_1 < 0$, then $f(\phi) = 0$ has a double real root and a single real root. Denote $F(\phi) = (\phi - \gamma_1)^2(\phi - \gamma_2)$, where $\gamma_1 \neq \gamma_2$. When $\phi > \gamma_2$, we have

$$\xi - \xi_0 = \int \frac{d\phi}{(\phi - \gamma_1)^2(\phi - \gamma_2)} = \begin{cases} \dfrac{1}{\sqrt{\gamma_1 - \gamma_2}} \ln\left|\dfrac{\sqrt{\phi - \gamma_2} - \sqrt{\gamma_1 - \gamma_2}}{\sqrt{\phi - \gamma_2} + \sqrt{\gamma_1 + \gamma_2}}\right|, & \gamma_1 > \gamma_2, \\ \dfrac{2}{\sqrt{\gamma_2 - \gamma_1}} \arctan\sqrt{\dfrac{\phi - \gamma_2}{\gamma_2 - \gamma_1}}, & \gamma_1 < \gamma_2. \end{cases} \tag{11}$$

Then, the solution of Equation (8) is

$$u_1(t,x) = \left(\frac{2a}{3b}\right)^{-1/3}\left\{(\gamma_1 - \gamma_2)\tanh^2 \cdot \left[\frac{\sqrt{\gamma_1 - \gamma_2}}{2}\left(\frac{2a}{3b}\right)^{1/3}\left(\frac{x^\beta}{\beta} - c\frac{t^\alpha}{\alpha} - \xi_0\right)\right] + \gamma_2\right\}, \gamma_1 > \gamma_2,$$

$$u_2(t,x) = \left(\frac{2a}{3b}\right)^{-1/3}\left\{(\gamma_1 - \gamma_2)\coth^2 \cdot \left[\frac{\sqrt{\gamma_1 - \gamma_2}}{2}\left(\frac{2a}{3b}\right)^{1/3}\left(\frac{x^\beta}{\beta} - c\frac{t^\alpha}{\alpha} - \xi_0\right)\right] + \gamma_2\right\}, \gamma_1 > \gamma_2,$$

$$u_3(t,x) = \left(\frac{2a}{3b}\right)^{-1/3} \left\{ (-\gamma_1 + \gamma_2) \tan^2 \cdot \left[\frac{\sqrt{-\gamma_1 + \gamma_2}}{2} \left(\frac{2a}{3b}\right)^{1/3} \left(\frac{x^\beta}{\beta} - c\frac{t^\alpha}{\alpha} - \xi_0 \right) \right] + \gamma_2 \right\}, \gamma_1 < \gamma_2. \tag{12}$$

Remark 3. When $a = 3$, $b = 1$, $c = \sqrt{5}$, $\beta = 1/3$, $c_1 = \sqrt{5}$, $c_2 = 3$, $c_3 = -2$, and $\xi_0 = 0$, three-dimensional diagram and two-dimensional diagram of the solution $u_1(t,x)$ of Equation (1) are drawn in Figure 1, respectively.

Case 2. If $\Delta = 0$ and $D_1 = 0$, then $f(\phi) = 0$ has a triple real root. Denoting $F(\phi) = (\phi - \gamma)^3$, then we have

$$u_4(t,x,z) = 4\left(\frac{2a}{3b}\right)^{-2/3} \left(\frac{x^\beta}{\beta} - c\frac{t^\alpha}{\alpha} - \xi_0 \right)^{-2} + \gamma. \tag{13}$$

Case 3. If $\Delta > 0$ and $D_1 < 0$, then $f(v) = 0$ has three different real roots, γ_1, γ_2, and γ_3, and $\gamma_1 < \gamma_2 < \gamma_3$.

If $\gamma_1 < \phi < \gamma_3$, taking the transformation $\phi = \gamma_1 + (\gamma_2 - \gamma_1)\sin^2\zeta$, then we obtain

$$\begin{aligned} \pm(\xi - \xi_0) &= \int \frac{d\phi}{\sqrt{f(\phi)}} \\ &= \int \frac{2(\gamma_2 - \gamma_1)\sin\zeta\cos\zeta d\zeta}{\sqrt{\gamma_3 - \gamma_1}(\gamma_2 - \gamma_1)\sin\zeta\cos\zeta\sqrt{1 - m^2\sin^2\zeta}} \\ &= \frac{2}{\sqrt{\gamma_3 - \gamma_1}} \int \frac{d\zeta}{\sqrt{1 - m^2\sin^2\zeta}}, \end{aligned} \tag{14}$$

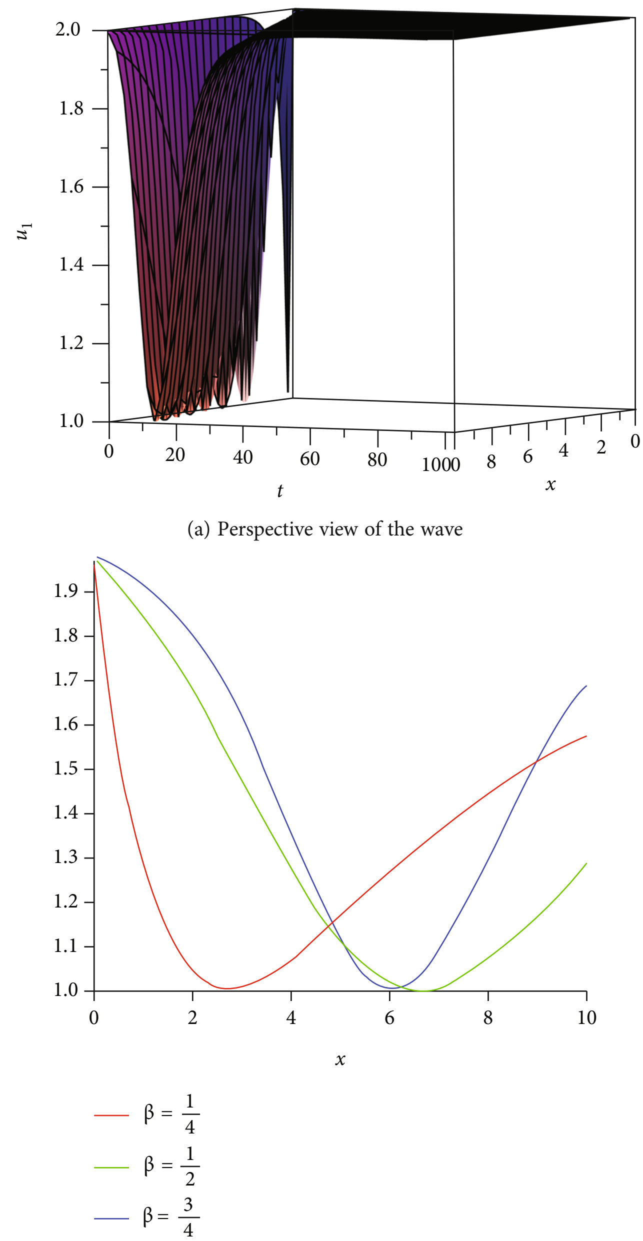
where $m^2 = \gamma_2 - \gamma_1/\gamma_3 - \gamma_1$.

From the definition of Jacobi function and (14), we have

$$\phi = \gamma_1 + (\gamma_2 - \gamma_1) sn^2 \left(\frac{\sqrt{\gamma_3 - \gamma_1}}{2}(\xi - \xi_0), m \right). \tag{15}$$

Then, the solution of the corresponding Equation (8) is

$$u_5(t,x) = \left(\frac{2a}{3b}\right)^{-1/3} \left[\gamma_1 + (\gamma_2 - \gamma_1) sn^2 \cdot \left(\frac{\sqrt{\gamma_3 - \gamma_1}}{2} \left(\frac{2a}{3b}\right)^{1/3} \left(\frac{x^\beta}{\beta} - c\frac{t^\alpha}{\alpha} - \xi_0 \right), m \right) \right]. \tag{16}$$



(a) Perspective view of the wave

(b) The wave along x

Figure 1: The solution $u_1(t,x)$ at $a = 3$, $b = 1$, $c = \sqrt{5}$, $\beta = 1/3$, $c_1 = \sqrt{5}$, $c_2 = 3$, $c_3 = -2$, and $\xi_0 = 0$.

If $\phi > \gamma_3$, take the transformation $\phi = -\gamma_2\sin^2\zeta + \gamma_3/\cos^2\zeta$. Similarly, the solution of the corresponding Equation (8) is

$$u_6(t,x) = \left(\frac{2a}{3b}\right)^{-1/3} \left[\frac{\gamma_3 - \gamma_2 sn^2\left(\sqrt{\gamma_3 - \gamma_1}/2(2a/3b)^{1/3}\left(x^\beta/\beta - c(t^\alpha/\alpha) - \xi_0\right), m\right)}{cn^2\left(\sqrt{\gamma_3 - \gamma_1}/2(2a/3b)^{1/3}\left(x^\beta/\beta - c(t^\alpha/\alpha) - \xi_0\right), m\right)} \right]. \tag{17}$$

where $m^2 = \gamma_2 - \gamma_1/\gamma_3 - \gamma_1$.

Case 4. If $\Delta < 0$, then $f(\phi) = 0$ has only real root. Denote $F(\phi) = (\phi - \gamma)(\phi^2 + p\phi + q)$, where $p^2 - 4q < 0$.

If $\phi > \gamma$, taking the transformation $\phi = \gamma + \sqrt{\gamma^2 + p\gamma + q}\tan^2\zeta/2$, then we obtain

$$\begin{aligned}\xi - \xi_0 &= \int \frac{d\phi}{\sqrt{(\phi-\gamma)(\phi^2 + p\phi + q)}} \\ &= \int \frac{\sqrt{\gamma^2 + p\gamma + q}\tan\zeta/2/\cos^2\zeta/2 d\zeta}{(\gamma^2 + p\gamma + q)^{3/4}\tan\zeta/2/\cos^2\zeta/2\sqrt{1 - m^2\sin^2\zeta}} \\ &= \frac{1}{(\gamma^2 + p\gamma + q)^{1/4}} \int \frac{d\zeta}{\sqrt{1 - m^2\sin^2\zeta}},\end{aligned} \tag{18}$$

where $m^2 = 1/2(1 - \gamma + p/2/\sqrt{\gamma^2 + p\gamma + q})$.

From the definition of Jacobi function and (18), we have $cn((\gamma^2 + p\gamma + q)^{1/4}(\xi - \xi_0), m) = \cos\zeta$.

$$\cos\zeta = \frac{2\sqrt{\gamma^2 + p\gamma + q}}{\phi - \gamma + \sqrt{\gamma^2 + p\gamma + q}} - 1. \tag{19}$$

Then, if $\phi > \gamma$, the solution of the corresponding Equation (8) is

$$\phi_7(\xi) = \gamma + \frac{2\sqrt{\gamma^2 + p\gamma + q}}{1 + cn\left((\gamma^2 + p\gamma + q)^{1/4}(\xi - \xi_0), m\right)} - \sqrt{\gamma^2 + p\gamma + q}. \tag{20}$$

Similarly, the solution of the corresponding Equation (8) is

$$u_7(t,x) = \left(\frac{2a}{3b}\right)^{-1/3}\left[\gamma + \frac{2\sqrt{\gamma^2 + p\gamma + q}}{1 + cn\left((\gamma^2 + p\gamma + q)^{1/4}(2a/3b)^{-1/3}\left(x^\beta/\beta - c(t^\alpha/\alpha) - \xi_0\right), m\right)} - \sqrt{\gamma^2 + p\gamma + q}\right]. \tag{21}$$

$$\begin{aligned}
v_1(t,x) &= \left(\frac{2a}{3b}\right)^{-1/3} c\left\{(\gamma_1 - \gamma_2)\tanh^2\left[\frac{\sqrt{\gamma_1 - \gamma_2}}{2}\left(\frac{2a}{3b}\right)^{1/3}\left(\frac{x^\beta}{\beta} - c\frac{t^\alpha}{\alpha} - \xi_0\right)\right] + \gamma_2\right\} + c_1, \gamma_1 > \gamma_2, \\
v_2(t,x) &= \left(\frac{2a}{3b}\right)^{-1/3} c\left\{(\gamma_1 - \gamma_2)\coth^2\left[\frac{\sqrt{\gamma_1 - \gamma_2}}{2}\left(\frac{2a}{3b}\right)^{1/3}\left(\frac{x^\beta}{\beta} - c\frac{t^\alpha}{\alpha} - \xi_0\right)\right] + \gamma_2\right\} + c_1, \gamma_1 > \gamma_2, \\
v_3(t,x) &= \left(\frac{2a}{3b}\right)^{-1/3} c\left\{(-\gamma_1 + \gamma_2)\tan^2\left[\frac{\sqrt{-\gamma_1 + \gamma_2}}{2}\left(\frac{2a}{3b}\right)^{1/3}\left(\frac{x^\beta}{\beta} - c\frac{t^\alpha}{\alpha} - \xi_0\right)\right] + \gamma_2\right\} + c_1, \gamma_1 < \gamma_2, \\
v_4(t,x,z) &= 4c\left(\frac{2a}{3b}\right)^{-2/3}\left(\frac{x^\beta}{\beta} - c\frac{t^\alpha}{\alpha} - \xi_0\right)^{-2} + c\gamma + c_1, \\
v_5(t,x) &= \left(\frac{2a}{3b}\right)^{-1/3} c\left[\gamma_1 + (\gamma_2 - \gamma_1)sn^2\left(\frac{\sqrt{\gamma_3 - \gamma_1}}{2}\left(\frac{2a}{3b}\right)^{1/3}\left(\frac{x^\beta}{\beta} - c\frac{t^\alpha}{\alpha} - \xi_0\right), m\right)\right] + c_1, \\
v_6(t,x) &= \left(\frac{2a}{3b}\right)^{-1/3} c\left[\frac{\gamma_3 - \gamma_2 sn^2\left(\sqrt{\gamma_3 - \gamma_1}/2(2a/3b)^{1/3}\left(x^\beta/\beta - c(t^\alpha/\alpha) - \xi_0\right), m\right)}{cn^2\left(\sqrt{\gamma_3 - \gamma_1}/2(2a/3b)^{1/3}\left(x^\beta/\beta - c(t^\alpha/\alpha) - \xi_0\right), m\right)}\right] + c_1, \\
v_7(t,x) &= \left(\frac{2a}{3b}\right)^{-1/3} c\left[\gamma + \frac{2\sqrt{\gamma^2 + p\gamma + q}}{1 + cn\left((\gamma^2 + p\gamma + q)^{1/4}(2a/3b)^{-1/3}\left(x^\beta/\beta - c(t^\alpha/\alpha) - \xi_0\right), m\right)} - \sqrt{\gamma^2 + p\gamma + q}\right] + c_1.
\end{aligned} \tag{22}$$

Remark 4. In the time-space fractional coupled Boussinesq equations, the unknown functions u and v satisfy the relationship $v = cu - c_1$. In the paper, the traveling wave solutions of Equation (1) have been obtained by the complete discrimination system method, and we can easily obtain the solutions of v of Equation (1) by using (6).

Remark 5. The solutions obtained in references [16, 17] mainly focus on hyperbolic function solutions and trigonometric function solutions. However, in this paper, not only the trigonometric function solution and hyperbolic function solution but also the Jacobian function solution and implicit function solution are obtained. Therefore, a new solution is obtained in this paper.

3. Conclusion

The fractional coupled Boussinesq equations, which are usually used to simulate nonlinear shallow water surface wave phenomena, are studied by the complete discrimination system method. A series of new exact solutions are obtained, including rational function solutions, Jacobian elliptic function solutions, implicit solutions, hyperbolic function solutions, and trigonometric function solutions. Compared with the existing literature [16, 17], the implicit function solutions and Jacobian function solutions obtained in the paper are new solutions. Moreover, the complete discrimination system method can also be used to find the exact traveling wave solutions of other coupled systems. In future research work, we will focus on the exact traveling wave solution of more complex coupled systems.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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