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Research Article

Hybrid Meson Interpretation of the Exotic Resonance $\pi_1(1600)$

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The exotic $J^{PC} = 1^{-+}$resonance $\pi_1(1600)$ is examined in the framework of the Quark Model with Constituent Gluon (QMCG). We report the possibility of interpreting that resonance as $q\bar{q}g$ meson, with a masse $\simeq 1.65^{+0.05}_{-0.04}$ GeV, and a decay width to $\rho\pi \simeq 0.28^{+0.14}_{-0.09}$ GeV.

1. Introduction

Over the last four decades, intensive experimental activity has been carried out seeking to detect new hadrons beyond the quark model: glueballs or gluonium, hybrids, diquonia, and tetraquarks. These "exotic" species are most likely the new hadrons allowed by the QCD and are the subject of numerous researches, both theoretical and experimental.

Hybrid mesons (quark-antiquark-gluon) can have J^{PC} quantum numbers which are not allowed by the naive quark model, like 0^{--}, 0^{+-}, 1^{-+}, and 2^{+-}, then they cannot mix with the standard mesons and hence can facilitate their observation. These "exotic" objects are the most promising new species of hadrons allowed by QCD and subject of lot of works both in the theoretical and experimental levels. In fact, several $J^{PC} = 1^{-+}$ exotic resonances have been claimed to be identified, especially $\pi_1(1600)$ and $\pi_1(1400)$ have received great interest, but some doubts are raised about the last one (for a review, see Ref [1]).

In the theoretical framework, these hybrid mesons were studied from different models: lattice QCD [2–7], flux tube model [8–11], bag model [12, 13], QCD sum rules [14–18], constituent gluon models [19–26], and the effective Hamiltonian model [11, 27, 28]. Some of these models can perform both estimations of masses and decay widths, they predicted that the lightest hybrid mesons will be in 1.4-2.1 GeV mass range which is consistent with the confirmed 1^{-+} candidates.

The $\pi_1(1600)$ was observed decaying into $b_1\pi$, $f_1\pi$, $\eta'\pi$, and $\rho\pi$. But although the first three modes have been confirmed for a long time, the $\rho\pi$ mode has been incorporated only recently in PDG since 2018 [29]. Indeed, this mode is forbidden due to the "standard" flux tube predictions in a symmetry limit where the ρ and π have the same size and in the case where the decay is triggered by breaking the flux tube [8–11], although a value of 57 MeV was calculated beyond this limit [30]. This remains quite far from the very recent measurements made by COMPASS experiment [31] (see also PDG-2018 [29]).

In this work, we focus our attention on the 1^{-+} hybrid meson in the context of the *Quark Model* with *Constituent Gluon* (QMCG), and we shall see that this constituent glue model gives values of the mass and the $\rho\pi$ decay width of the lightest 1^{-+} $q\bar{q}g$ quite compatible with the observed exotic candidate $\pi_1(1600)$.

This paper is organized as follows. In Section 2, we briefly present the experimental situation of the exotic $J^{PC} = 1^{-+}\pi_1$ (1600). We give predictions of the model QMGC in Section 3, and we conclude in Section 4.

2. The Experiemental Status of $\pi_1(1600)$

We consider here only the status of the resonance $\pi_1(1600)$ (for a review of the experimental situation on the exotic

Table 1: The $\pi_1(1600)$ as seen in different experiments. The resonance masses and the corresponding decay widths are reported in GeV.

	$\rho\pi$	$b_1\pi$	$f_1(1235)\pi$	$\eta'\pi$
VES	~1.6 $0.3 * 0.34 = 0.10$ [a]	1.56 ± 0.06 0.34 ± 0.06	1.64 ± 0.03 0.24 ± 0.06	1.56 ± 0.06 0.34 ± 0.06
E852	$1.593 \pm 0.008^{+0.029}_{-0.047}$ $0.168 \pm 0.020^{+0.150}_{-0.012}$ [b]	$1.664 \pm 0.008 \pm 0.010$ $0.185 \pm 0.025 \pm 0.028$	$1.709 \pm 0.024 \pm 0.041$ $0.403 \pm 0.080 \pm 0.115$	$1.597 \pm 0.010^{+0.045}_{-0.010}$ $0.340 \pm 0.040 \pm 0.060$
COMPASS	$1.600^{+0.100}_{-0.060}$ $0.580^{+0.100}_{-0.230}$	—	—	$1.564 \pm 0.024 \pm 0.086$ $0.492 \pm 0.054 \pm 0.102$ [c]
CLEO	—	—	—	$1.670 \pm 0.030 \pm 0.020$ $0.240 \pm 0.050 \pm 0.060$
Crystal Barrel	—	~1.6 seen	—	—

[a]An experimental decay width limit. [b]This state is excluded by a more refined E852 analysis (Ref. [38]). [c]From JLAC collaboration [45] using COMPASS data [44].

hybrid mesons, see the Ref [1] that we have mainly used when preparing this section).

VES Collaboration [32] observed a broad peak at a mass value of ~1.6 GeV in the $\eta'\pi$, $f_1(1235)\pi$, and $b_1(1235)\pi$ systems, interpreted as an exotic resonance of width about 300 MeV. Actually, they are unable to make a definitive conclusion on the resonance nature of it. For the $\rho\pi$ final state, they are unable to conclude that the $\pi_1(1600)$ is present, while the following experimental relationship between the branching fractions of the $\pi_1(1600)$ decays is obtained (and therefore a limit on the branching fraction of $\rho\pi$):

$$b_1\pi : f_1\pi : \rho\pi : \eta'\pi = (1.0 \pm 0.3) : (1.1 \pm 0.3) : < 0.3 : 1. \quad (1)$$

E852 Collaboration at BNL reported an evidence for the 1^{-+} $\pi_1(1600)$ resonance decaying into $\rho\pi$ [33, 34], $\eta'\pi$ [35], $f_1\pi$ [36], and $b_1\pi$ [37], regarding the $\rho\pi$ channel, in the earlier E852 analysis [33, 34] of 250 K $(\pi^-\pi^-\pi^+)$ events showed a possible evidence for a 1^{-+} exotic meson with a mass of ~1.6 GeV and width ~168 MeV, this state have been excluded by a more refined analysis [38], with 2.6 M $(\pi^-\pi^-\pi^+)$ and 3 M $(\pi^-\pi^0\pi^0)$ events.

The exotic $\pi_1(1600)$ is observed decaying to $b_1\pi$ from the Crystal Barrel data, and only results with mass and width fixed to the PDG values were reported [39].

CLEO Collaboration found evidence for an exotic P-wave $\eta'\pi$ amplitude, which, if interpreted as a resonance, would have parameters consistent with the $\pi_1(1600)$ state with a mass of $1670 \pm 30 \pm 20$ MeV and a width of $240 \pm 50 \pm 60$ MeV [40].

A search for exotic mesons in the $(\pi^+\pi^+\pi^-)$ system photoproduced by the charge exchange reaction $\gamma p \longrightarrow \pi^+\pi^+\pi^-(n)$ was carried out by the CLAS Collaboration at Jefferson Lab., and no evidence is shown of the exotic $\pi_1(1600)$ decaying to three charged pions [41, 42].

COMPASS collaboration observed the spin-exotic $\pi_1(1600)$ in their partial-wave analysis of the 3π final state. They reported the observation of the $\pi_1(1600)$ in the $\rho\pi$ decay mode initially with a mass $1660 \pm 10^{+0}_{-64}$ MeV and width $269 \pm 21^{+42}_{-64}$ MeV [43], superseded by a mass $1.600^{+0.100}_{-0.060}$ GeV and width $0.580^{+0.100}_{-0.230}$ GeV in their recent analysis [31]. COMPASS collaboration has also examined the exclusive production of $\eta\pi$ and $\eta'\pi$ and reported that odd partial waves, which carry non-$q\bar{q}$ quantum numbers, are suppressed in the $\eta\pi$ system relative to the $\eta'\pi$ system. Even though they saw the exotic 1^{-+} wave in $\eta'\pi$ as the dominant wave, they were unable to confirm the resonant nature of the signal [44]. This has recently been improved by the JPAC collaboration [45] which performed the first coupled-channel analysis of the P-wave in the $\eta^{(\prime)}\pi$ system measured at COMPASS [44] and reported a single exotic π_1 with mass and width determined to be $1564 \pm 24 \pm 86$ and $492 \pm 54 \pm 102$ MeV, respectively.

In conclusion, the $\pi_1(1600)$ was observed decaying into $b_1\pi$, $f_1\pi$, and $\eta'\pi$ and recently confirmed for $\rho\pi$ mode by COMPASS collaboration [31], it is considered by the PDG to be an established state [29]. Table 1 shows masses and the corresponding decay widths of the $\pi_1(1600)$ reported by different experiments.

3. The QMCG Predictions

The nature of the gluonic field inside the hybrid meson is not yet clear because the gluon plays a double role: it propagates the interaction between color sources and, being itself colored, it undergoes the interaction. In an attempt to achieve a clearer understanding about the hybrid nature, two important hypothesis can be retained from the literature. The first one consider gluonic degrees of freedom as "excitations" of the "flux tube" between quark and antiquark, which leads to the linear potential, that is familiar from the quark model (flux tube model).

In the second one, the framework of the so-called *Quark Model* with *Constituent Gluon* (QMCG) supported by this work, the hybrid meson is considered as a QCD-bound state composed of a quark-antiquark pair and (a massive) constituent gluon which interact through a phenomenological potential. We can adapt this scheme with the idea of confined and confining gluons (in the Landau and Coulomb gauges and in interpolating gauges between them) [46]. Confining gluons establish an area law behavior of the Wilson loop and the linearly rising interquark confinement, while confined gluons do not propagate over long distances, we can accommodate the confined (massive, constituent) gluon in coexistence with an effective quark interaction which is confining (more details can be found in Ref [24, 25]).

"It is important to realise that the more complicated picture emerging for QCD in the covariant gauge can certainly accommodate confined (but not confining) gluons in coexistence with an effective quark interaction which is confining, however." [47].

3.1. Ingredients of the QMGC. The QMCG is a natural expansion of the naive quark model where the confined gluon within the hadron matter acquires a (constituent) mass m_g. As for quarks, this important parameter represents a dynamical mass which is responsible for the infrared finiteness of the gluon propagator and the ghost dressing function observed using continuum methods (the Schwinger-Dyson Equations) and large-volume lattice simulations or combining continuum methods with lattice data (a more complete presentation of the subject is given for example in Refs. [48–51]).

From the phenomenological point of view, a nonvanishing gluon mass is welcome by diffractive phenomena [52] and inclusive radiative decays of J/ψ and Υ [53]. For the glueball states, color singlet bound states of gluons are considered to be fairly massive, e.g., about 1.5 GeV for the lowest 0^{++} and about 2 GeV for the lowest 2^{++}, as indicated in lattice QCD calculations [54–56], a simple constituent gluon picture may be approximately obtained as $M_{\mathrm{GB}} \simeq 2m_g$ for the glueball mass M_{GB}.

Using the continuum strong QCD, one infers $m_g \simeq 0.4 - 0.6\,\mathrm{GeV}$ [51] which is consistent with the lattice results: $m_g \sim 0.5\,\mathrm{GeV}$ [57, 58].

In the present work, we fix this parameter as:

$$m_g = 0.5 \pm 0.1\,\mathrm{GeV}. \tag{2}$$

The decay parameter α_s (the effective quark-gluon vertex coupling) is the second ingredient of the model. There are many theoretical evidences that the QCD effective charge α_s freezes at small momenta. Therefore, the infrared finiteness of the effective charge can be considered as one of the manifestations of the phenomenon of dynamical gluon mass generation. Phenomenology sensitive to infrared properties of QCD gives $\alpha_s(0) \simeq 0.7 \pm 0.3$ [59–61], while the phenomenological evidences for the strong coupling constant freezing in the infrared are much more numerous, as with models where a static potential is used to compute the hadronic spectra that make use of a frozen coupling constant at long distances (for more details, see for example the Ref. [62]).

The effective charge obtained within the pinch technique (PT) framework [63, 64], to be denoted by α_{PT}, constitutes the most direct non-Abelian generalization of the familiar concept of the QED effective charge. Since our decay model is obtained in the Feynman gauge [20], it is natural to choose $\alpha_s \simeq \alpha_{\mathrm{PT}}(0)$ corresponding to the pinch technique gluon propagator, i.e., the background field propagator calculated in the Feynman gauge. $\alpha_{\mathrm{PT}}(0)$ is correlated to the gluon mass m_g [65, 66]:

$$\alpha_{\mathrm{PT}}(0) \sim 0.6 \text{ for } m_g \sim 0.5\,\mathrm{GeV}. \tag{3}$$

3.2. The Hybrid Bound State. We assume that the hybrid meson is a bound state of quark-antiquark and a constituent gluon which interact through a phenomenological potential, precisely Coulomb plus linear potential supplemented by spin-spin, spin-orbit, and tensor correction terms. The use of relativistic kinetics is appropriate for the study of the light flavor systems [24].

For the representation of the hybrid states, the following notations are used:

(1) l_g: the relative orbital momentum of the gluon in the $q\bar{q}$ center of mass

(2) $l_{q\bar{q}}$: the relative orbital momentum between q and $\bar{q}$

(3) $S_{q\bar{q}}$: the total quarks spin

Considering the gluon moving in the framework of the $q\bar{q}$ pair, the Parity of the hybrid will be:

$$P = (-)^{l_{q\bar{q}}+1} \cdot (-1) \cdot (-)^{l_g} = (-)^{l_{q\bar{q}}+l_g}, \tag{4}$$

(-1) being the intrinsic parity of the gluon.

The Charge Conjugation is given by:

$$C = (-)^{l_{q\bar{q}}+S_{q\bar{q}}+1}. \tag{5}$$

$S_{q\bar{q}}$ can take the values 0 or 1; P and C impose parity restrictions on $l_{q\bar{q}}$ and l_g.

For lower values of the orbital excitations ($l_{q\bar{q}}$ and $l_g \leqslant 1$) and parity $P = -1$, the hybrid states can be built by two modes: $l_{q\bar{q}} = 0$ and $l_g = 1$ which we shall refer as the gluon-excited hybrid (GE hybrid), and $l_{q\bar{q}} = 1$ and $l_g = 0$ which we shall refer as the quark-excited hybrid (QE hybrid) (see Table 2 for the case $J^{\mathrm{PC}} = 1^{-+}$).

In the potential model, the simplest approximation is to factorise the $q\bar{q}$-wave function with the wave function of the gluon respective to the $q\bar{q}$ center of mass (the cluster

Table 2: The lowest $J^{PC} = 1^{-+}$ hybrid meson quantum numbers. Modes and the decay selection rules are shown.

$l_{q\bar{q}}$	$S_{q\bar{q}}$	l_g	j_g	L	Hybrid mode	Gluon mode	Preferred decay mode
0	1	1	0	0	GE	Long.	"$L+S$"
0	1	1	1	1	GE	Long., TM, TE	"$L+S$"
0	1	1	2	2	GE	Long., TM, TE	"$L+S$"
1	0	0	1	1	QE	Long., TM	"$S+S$"

approximation). We shall use the following lowest-lying state $q\bar{q}$-cluster spin-space wave function:

$$\Psi_{JM}^{PC}\left(\vec{\rho},\vec{\lambda}\right) = \left(\left(\left(\mathbf{e}_{\mu_g} \otimes \psi_{l_g}^{m_g}\right)_{j_g M_g} \otimes \psi_{l_{q\bar{q}}}^{m_{q\bar{q}}}\right)_{Lm} \otimes \chi_{\mu_{q\bar{q}}}^{S_{q\bar{q}}}\right)_{\mathrm{JM}}^{\mathrm{PC}} = \sum_{\left(L l_g j_g l_{q\bar{q}} S_{q\bar{q}};\mathrm{PC}\right)} \Psi_{\mathrm{JM};L l_g j_g l_{q\bar{q}} S_{q\bar{q}}}\left(\vec{\rho},\vec{\lambda}\right), \quad (6)$$

where $\mathbf{e}_{\mu_g}$ is the gluon polarisation, $\chi_{\mu_{q\bar{q}}}^{S_{q\bar{q}}}$ is the diquark spin representation, and the sum runs over the values of L, l_g, j_g, $l_{q\bar{q}}$ and $S_{q\bar{q}}$ excluding those not consistent with P and C and:

$$\Psi_{\mathrm{JM};L l_g j_g l_{q\bar{q}} S_{q\bar{q}}}\left(\vec{\rho},\vec{\lambda}\right) = \sum_{\left(m\, m_g \mu_g M_g m_{q\bar{q}} \mu_{q\bar{q}}\right)} \left\langle l_g m_g \mathbf{1} \mu_g \mid j_g M_g \right\rangle \times \left\langle l_{q\bar{q}} m_{q\bar{q}} j_g M_g \mid Lm \right\rangle \left\langle L m S_{q\bar{q}} \mu_{q\bar{q}} \mid \mathrm{JM} \right\rangle \times \psi_{l_g l_{q\bar{q}}}^{m_g m_{q\bar{q}}}\left(\vec{\rho},\vec{\lambda}\right) \mathbf{e}_{\mu_g} \chi_{\mu_{q\bar{q}}}^{S_{q\bar{q}}}; \quad (7)$$

here, the Jacobi coordinates are introduced:

$$\vec{\rho} = \vec{r}_{\bar{q}} - \vec{r}_q, \qquad \vec{\lambda} = \vec{r}_g - \frac{M_q \vec{r}_q + M_{\bar{q}} \vec{r}_{\bar{q}}}{M_q + M_{\bar{q}}}. \quad (8)$$

The Hamiltonian is constructed, containing a phenomenological potential which reproduces the QCD characteristics; its expression has the mathematical "Coulomb + Linear" form, we take into account also some relativistic effects, i.e., spin-dependent interaction terms and relativistic kinetics; a more detailed description can be found in our previous work [24].

In order to make a comparison with the lattice results, we note that our 1^{-+} wave function (Eq. (7)) is related to the so-called TE, TM, and longitudinal gluon states as follows (see Table 2):

$$\Psi_{1M}^{-+} = \Psi_{j_g=0}^{\mathrm{long}} + \Psi_{j_g=1}^{\mathrm{TE}} + \mathrm{mix}\left(\Psi_{j_g=2}^{\mathrm{TM}}, \Psi_{j_g=2}^{\mathrm{long}}\right) + \mathrm{mix}\left(\Psi_{j_g=1}^{\mathrm{TM}}, \Psi_{j_g=1}^{\mathrm{long}}\right), \quad (9)$$

where $\mathrm{mix}(\varphi,\psi)$ means a mixture of the states φ and ψ. The "magnetic" TE, "electric" TM, and longitudinal gluons correspond to the following hybrid states [20]:

$$\Psi_{j_g}^{\mathrm{TE}} \equiv \Psi_{\mathrm{JM};L l_g j_g l_{q\bar{q}} S_{q\bar{q}}}^{\mathrm{TE}} = \Psi_{\mathrm{JM};L l_g j_g l_{q\bar{q}} S_{q\bar{q}}}\Big|_{l_g=j_g},$$
$$\Psi_{j_g}^{\mathrm{TM}} \equiv \Psi_{\mathrm{JM};L l_g j_g l_{q\bar{q}} S_{q\bar{q}}}^{\mathrm{TM}} = \sqrt{\frac{j_g+1}{2j_g+1}}\Psi_{\mathrm{JM};L l_g j_g l_{q\bar{q}} S_{q\bar{q}}}\Big|_{l_g=j_g-1} + \sqrt{\frac{j_g}{2j_g+1}}\Psi_{\mathrm{JM};L l_g j_g l_{q\bar{q}} S_{q\bar{q}}}\Big|_{l_g=j_g+1},$$
$$\Psi_{j_g}^{\mathrm{long}} \equiv \Psi_{\mathrm{JM};L l_g j_g l_{q\bar{q}} S_{q\bar{q}}}^{\mathrm{long}} = -\sqrt{\frac{j_g}{2j_g+1}}\Psi_{\mathrm{JM};L l_g j_g l_{q\bar{q}} S_{q\bar{q}}}\Big|_{l_g=j_g-1} + \sqrt{\frac{j_g+1}{2j_g+1}}\Psi_{\mathrm{JM};L l_g j_g l_{q\bar{q}} S_{q\bar{q}}}\Big|_{l_g=j_g+1}. \quad (10)$$

We can rewrite Eq. (9) according to the GE and QE hybrid modes as:

$$\Psi_{1M}^{-+} = \psi^{\mathrm{GE}} + \psi^{\mathrm{QE}}, \quad (11)$$

where

$$\psi^{\mathrm{GE}} = \Psi_{j_g=0}^{\mathrm{long}} + \Psi_{j_g=1}^{\mathrm{TE}} + \mathrm{mix}\left(\Psi_{j_g=2}^{\mathrm{TM}}, \Psi_{j_g=2}^{\mathrm{long}}\right), \quad (12)$$

$$\psi^{\mathrm{QE}} = \mathrm{mix}\left(\Psi_{j_g=1}^{\mathrm{TM}}, \Psi_{j_g=1}^{\mathrm{long}}\right). \quad (13)$$

Since our gluon is assumed to be massive, the longitudinal component must be present in Eq. (9), mixed with the TM and TE gluon modes $(j_g \neq 0)$. This is not true in the lattice hybrid calculations where the low-laying 1^{-+} states are made with the particular $j_g^{P_g C_g} = 1^{+-}$ TE-gluon mode. Indeed, although in principal lattice construction of the hybrid 1^{-+} states involves the TM and the TE modes, in the light sector, only the last mode results are widely reported since it gives the best and the clearest signal [2–5]. From Table 2, we notice that the TE gluon appears only in the GE-hybrid and is totally absent in the QE-hybrid 1^{-+} state. We will come back to this later.

3.3. The Hybrid Decay Model. To the lowest order, the decay of a hybrid state A into two ordinary mesons B and C is described by the matrix element of the Hamiltonian annihilating a gluon and creating a quark pair (QPC model):

$$\begin{aligned} H &= g\int d\vec{x}\,\bar{\psi}\left(\vec{x}\right)\gamma_\mu \frac{\lambda^a}{2}\psi\left(\vec{x}\right)A_a^\mu\left(\vec{x}\right) \\ &= g\sum_{ss'\lambda}\int \frac{d\vec{p}\,d\vec{k}\,d\vec{p'}}{\sqrt{2\omega}(2\pi)^9}(2\pi)^3\delta^{(3)}\left(\vec{p}-\vec{k}-\vec{p'}\right) \\ &\quad\times \bar{u}_{\vec{p}s}\gamma_\mu\frac{\lambda^a}{2}u_{-\vec{p'}s}{}'\varepsilon^\mu_{\vec{k}\lambda}\varphi_a b^\dagger_{\vec{p}s}d^\dagger_{-\vec{p'}s'}a_{\vec{k}\lambda}, \end{aligned} \tag{14}$$

where a is the color index and s, s', and λ are the spin indices. In the nonrelativistic limit, we have:

$$\bar{u}_{\vec{p}s}\gamma_\mu\frac{\lambda^a}{2}u_{-\vec{p'}s}\,\varepsilon^\mu_{c} \simeq \chi_s^\dagger\vec{\sigma}\bar{\chi}_s{}'\vec{\varepsilon}_{\vec{k}\lambda}, \tag{15}$$

where $\chi_s{}'$ is the antiquark spinor in the complex conjugate representation.

The standard meson ($|B\rangle$ or $|C\rangle$) and the hybrid ($|A\rangle$) states are written in the nonrelativistic approximation:

$$\begin{aligned} |q\bar{q};\mathrm{JM}\rangle &= \sum_{ss'a\bar{a}m\mu}\int\frac{d\vec{p}_q\,d\vec{p}_{\bar{q}}}{(2\pi)^6}(2\pi)^3\delta^{(3)} \\ &\quad\times\left(\vec{p}-\vec{p}_q-\vec{p}_{\bar{q}}\right)\langle lmS\mu\,|\,\mathrm{JM}\rangle \\ &\quad\times\frac{1}{\sqrt{3}}\chi^\mu_{ss'}\psi_{lm}\left(\frac{m_{\bar{q}}\vec{p}_q-m_q\vec{p}_{\bar{q}}}{m_{\bar{q}}+m_q}\right)b^\dagger_{\vec{p}_q sa}d^\dagger_{\vec{p}_{\bar{q}}s'\bar{a}}|0\rangle \end{aligned} \tag{16}$$

with $\int (d\vec{p}/(2\pi)^3)|\psi_{lm}(\vec{p})|^2 = 1$;

$$\begin{aligned} |q\bar{q}g;\mathrm{JM}\rangle &= \sum_{ss'a\bar{a}(m\mu)}\int\frac{d\vec{p}_q\,d\vec{p}_{\bar{q}}\,d\vec{k}}{(2\pi)^3}(2\pi)^3\delta^{(3)}\left(\vec{p}-\vec{p}_q-\vec{p}_{\bar{q}}-\vec{k}\right) \\ &\quad\times\frac{\lambda^{c_g}_{c_q c_{\bar{q}}}}{4}\chi^\mu_{ss'}\psi^{m_g m_{q\bar{q}}}_{l_g l_{q\bar{q}}} \\ &\quad\times\left(\frac{m_{\bar{q}}\vec{p}_q-m_q\vec{p}_{\bar{q}}}{m_{\bar{q}}+m_q},\frac{\left(m_{\bar{q}}+m_q\right)\vec{k}-m_g\left(\vec{p}_q+\vec{p}_{\bar{q}}\right)}{m_{\bar{q}}+m_q+m_g}\right) \\ &\quad\times\left\langle l_g m_g \mathbf{1}\mu_{\mathbf{g}}\,|\,j_g M_g\right\rangle\left\langle l_{q\bar{q}}m_{q\bar{q}}j_g M_g\,|\,Lm\right\rangle \\ &\quad\times\left\langle LmS_{q\bar{q}}\mu_{q\bar{q}}\,|\,\mathrm{JM}\right\rangle b^\dagger_{\vec{p}_q sa}d^\dagger_{\vec{p}_{\bar{q}}s'\bar{a}}a^\dagger_{\vec{k}\lambda}|0\rangle, \end{aligned} \tag{17}$$

where c_q, $c_{\bar{q}}$, and c_g are the color charge of the quark, antiquark, and gluon with $c_q, c_{\bar{q}} = 1, 2, 3$ and $c_g = 1, \cdots, 8$. We have also $\int (d\vec{p}\,d\vec{k}/(2\pi)^6)|\psi^{m_g m_{q\bar{q}}}_{l_g l_{q\bar{q}}}(\vec{p},k)|^2 = 1$.

The matrix element between a hybrid state A and two standard mesons B and C is given by:

$$\langle \mathrm{BC}\,|\,H\,|\,A\rangle = g\,f(A,B,C)\,(2\pi)^3\delta^{(3)}\left(\vec{p}_A-\vec{p}_B-\vec{p}_C\right), \tag{18}$$

where $f(A,B,C)$ is the decay amplitude:

$$\begin{aligned} f(A,B,C) &= \sum_{(m),(\mu)}\Phi\Omega X\left(\mu_{q\bar{q}},\mu_g;\mu_B,\mu_c\right)I\left(m_{q\bar{q}},m_g;m_B,m_c,m\right) \\ &\quad\times\left\langle l_g m_g 1\mu_g\,|\,J_g M_g\right\rangle\left\langle l_{q\bar{q}}m_{q\bar{q}}J_g M_g\,|\,Lm'\right\rangle\left\langle Lm'S_{q\bar{q}}\mu_{q\bar{q}}\,|\,\mathrm{JM}\right\rangle \\ &\quad\times\langle l_B m_B \mathrm{S_B}\mu_\mathrm{B}\,|\,J_B M_B\rangle\langle l_C m_C \mathrm{S_C}\mu_\mathrm{C}\,|\,J_C M_C\rangle. \end{aligned} \tag{19}$$

The amplitude f involves the flavor (Φ), the color (Ω), the nonrelativistic spin (X), and the spatial (I) overlaps defined as follows.

$$\Phi = \sqrt{(2I_B+1)(2I_C+1)1(2I_A+1)}\begin{Bmatrix} i_1 & i_3 & I_B \\ i_2 & i_4 & I_C \\ I_A & 0 & I_A \end{Bmatrix}\eta\varepsilon, \tag{20}$$

$I's(i's)$ label the hadron (quark) isospins, $\eta = 1$ if the gluon goes into strange quarks and $\eta = \sqrt{2}$ if it goes into nonstrange ones. ε^2 is the number of diagrams contributing to the decay. Indeed, one can check that two diagrams contribute with the same sign and magnitude for C, P, and G-Parity allowed decays while they cancel for forbidden ones. In the case of two identical final particles $\varepsilon = \sqrt{2}$. The term between brackets in Eq. (20) is the $9j$ symbol.

$$\begin{aligned} \Omega &= \frac{1}{24}\sum_{\mathrm{a}}\mathrm{Tr}(\lambda^{\mathrm{a}})^2 = \frac{2}{3}. \\ X\left(\mu_{q\bar{q}},\mu_g;\mu_B,\mu_c\right) &= \sum_s\sqrt{2}\sqrt{(2S_B+1)(2S_C+1)3\left(2S_{q\bar{q}}+1\right)} \\ &\quad\times\begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & S_B \\ \frac{1}{2} & \frac{1}{2} & S_C \\ S_{q\bar{q}} & 1 & S \end{Bmatrix} \\ &\quad\times\langle S_B\mu_B S_C\mu_C\,|\,S(\mu_B+\mu_C)\rangle \\ &\quad\times\left\langle S_{q\bar{q}}\mu_{q\bar{q}}1\mu_g\,|\,S\left(\mu_{q\bar{q}}+\mu_g\right)\right\rangle. \end{aligned} \tag{21}$$

Table 3: The 1^{-+} hybrid masses and the decay widths (in GeV) from the QMGC model.

Constituent glue model (this work)			
m_g	0.4	0.5	0.6
$M_{1^{-+}}$	1.61	1.65	1.70
$\Gamma(1^{-+} \longrightarrow \rho\pi)$	0.19	0.28	0.42
$\Gamma(1^{-+} \longrightarrow b_1\pi)$	0.19	0.29	0.46
$\Gamma(1^{-+} \longrightarrow f_1(1235)\pi)$	0.06	0.10	0.17

Finally, the spatial overlap is represented by the term:

$$I(m_{q\bar{q}}, m_g ; m_B, m_c, m) = \int \frac{d\vec{p}\, d\vec{k}}{(2\pi)^6} \psi^{m_g m_{q\bar{q}}}_{l_g l_{q\bar{q}}}\left(\vec{p}_B - \vec{p}, \vec{k}\right) \psi^{*}_{l_B m_B} \times \left(\frac{m_{\bar{q}_i} \vec{p}_B}{m_{\bar{q}_i} + m_q} - \vec{p} - \frac{1}{2}\vec{k}\right) \times \psi^{*}_{l_C m_C}\left(-\frac{m_{q_i} \vec{p}_B}{m_{q_i} + m_q} + \vec{p} - \frac{1}{2}\vec{k}\right) \times d\Omega_B\, Y^{*}_{lm}(\Omega_B), \tag{22}$$

where l, m label the orbital momentum between the two final mesons.

The partial width is given by:

$$\Gamma_{A\to BC} = 4\alpha_s |f(A, B, C)|^2 \frac{P_B E_B E_C}{M_A}, \tag{23}$$

where α_s represents the infrared quark-gluon vertex coupling. For more details on the decay model, see Refs. [20, 24]; here, we focus on the main (nonrelativistic) results:

(1) The 1^{-+} QE-hybrid is allowed to decay into two S-wave mesons only (the so-called "$S + S$" selection rule)

(2) The 1^{-+} GE-hybrid is allowed to decay into a channel with one S-wave meson and one P-wave meson only (the so-called "$L + S$" selection rule)

The last selection rule is also reported in the gluonic excitation models of hybrid where the decay to two S-wave mesons is strongly suppressed (see [67] and references therein).

In the decay model which we use, the $\eta^{(\prime)}\pi$ decay modes are suppressed by the nonrelativistic spin conservation law although the spatial overlap is not vanishing for the QE-hybrid mode, a full relativistic studies shall give nonvanishing answer for both QE and GE modes (this will be the subject of future work). In the other side, it seems that the flux tube model [8] and the QCD sum rules [15] predict a suppression of 1^{-+}hybrid $\longrightarrow \eta\pi$. This is confirmed using a quite independent model way without any further hypothesis than the quenched approximation [68]. However, this approximate selection rule is related only to the "magnetic" or TE-gluon mode.

4. Results and Discussion

4.1. The Mass Results. Our results related to the 1^{-+} hybrid masses and decay widths for $m_g = 0.4 - 0.6$ GeV are summarised in Tables 3 and 4, we add Table 5 for comparison purposes.

It is difficult to get a hybrid masse lower than 1.5 GeV ($\leq$1.52 GeV for $m_g \geq 0.$).

We observe a large mixing between the two QE and GE-hybrid modes where all the TE, TM, and longitudinal gluon modes are included in the hybrid wave function (Eq. (9)); for a pure GE-mode (with excited glue $l_g = 1$ and an S-wave $q\bar{q}$), we have $M^{\text{GE}}_{1^{-+}} \simeq 1.76 \pm 0.05$ GeV for $m_g = 0.5 \pm 0.1$ GeV.

Our calculated mass is:

$$M_{1^{-+}} \simeq 1.65^{+0.05}_{-0.04}\ \text{GeV}, \tag{24}$$

which is very close to the latest PDG average [29]:

$$1.660^{+0.015}_{-0.011}\ \text{GeV}, \tag{25}$$

and quite far from ~ 2. GeV emerged from the lattice QCD [6] and the flux tube [9] studies that systematically discard the QE-mode where the gluon is not excited, i.e., ignore states electric TM ($j_g = 1$) and longitudinal (Eq. (13)). In addition, there is some difficulties that taint the lattice masse calculations:

(i) How to identify interpolation fields used as a hybrid and distinguish them from ordinary mesons? From the criteria for hybrids proposed in [5] (and adopted implicitly by earlier lattice works [2–4]) the hybrid-like character is directly related to the overlap with the appropriate J^{PC} interpolating fields. This is not always true, we cannot understand the nature of a state by the appearance of its interpolation field. This is sufficiently illustrated by the strong projection on η and η' produced with the glue interpolation field $G_{\mu\nu}\tilde{G}_{\mu\nu}$, it does not mean that they are glueballs [69]

(ii) In the light sector, lattice authors report only results related to the $j_g^{P_g C_g} = 1^{+-}$ TE-gluon since it has the best signal with the smallest statistical errors while the explicit masses of the $j_g^{P_g C_g} = 1^{--}$ TM-gluon are not yet published

(iii) The lattice calculation sill uses an unrealistic mass of the π meson (~ 396 MeV) which is much greater than the observed one (~ 139 MeV).

Table 4: Our 1^{-+} hybrid masses and the decay widths (in GeV) compared to the lattice QCD and flux tube results.

	Constituent glue model (this work)			Flux tube model		Lattice QCD
m_g=0.5 GeV	QE-GE mix.	QE	GE	Ref. [9]	Ref. [11]	Ref. [7]
$\Gamma(1^{-+}\text{hybrid} \longrightarrow \rho\pi)$	0.28	0.28	Forbidden[a]	0.005-0.020	0.009	—
$\Gamma(1^{-+}\text{hybrid} \longrightarrow b_1\pi)$	0.29	Forbidden[b]	0.29	0.170	0.024	0.40 ± 0.12
$\Gamma(1^{-+}\text{hybrid} \longrightarrow f_1(1235)\pi)$	0.10	Forbidden[c]	0.10	0.060	0.005	0.09 ± 0.06
$\Gamma\left(1^{-+}\text{hybrid} \longrightarrow \eta\pi, \eta'\pi\right)$	—	Forbidden[d]	<0.010	0.000-0.010	0.000	—

[a]Selection rule. [b]Selection rule. [c]Selection rule. [d]Allowed beyond the nonrelativistic approximation. Calculations of this contribution will be the subject of future work.

Table 5: The comparison of our work with data and the recent gluonic excitation models.

This work		The experience				Gluonic excitation models	
m_g	0.5±0.1	VES	E852	CLEO	COMPASS	Lattice QCD	Flux tube[a]
$M_{1^{-+}}$	$1.65^{+0.05}_{-0.04}$	1.64 ± 0.03	1.66 ± 0.01	1.67 ± 0.04	$1.60^{+0.10}_{-0.06}$	1.79 ± 0.14[b], 1.96 ± 0.04[c]	~200.
$\rho\pi$	$0.28^{+0.14}_{-0.09}$	$\lesssim 0.10$[d]	$0.17 \pm 0.02^{+0.15}_{-0.01}$	—	$0.58^{+0.10}_{-0.23}$	—	0.005 – 0.020
$b_1\pi$	$0.29^{+0.16}_{-0.11}$	0.34 ± 0.06	0.19 ± 0.04	—	—	0.40 ± 0.12[e]	0.170
$f_1(1235)\pi$	$0.10^{+0.07}_{-0.04}$	0.24 ± 0.06	0.40 ± 0.14	—	—	0.09 ± 0.06[f]	0.060

[a]Ref. [9]. [b]Ref. [6]. [c]From Figure 5 of Ref. [5] at $m_\pi = 396$ MeV ($\ggg$139 MeV, the observed value). [d]An experimental limit. [e]Ref. [7]. [f]Ref. [7].

4.2. The Decay Results. As shown in Eq. (23), the decay width is proportional to the parameter α_s which is in turn correlated to the mass of the gluon m_g, as mentioned above. In Table 3, we represent the results for $m_g = 0.5 \pm 0.1$ GeV and the final theoretical uncertainty is taken as the deviation from the nominal value and the upper and lower tolerance.

Despite the imperfections of the model, our predictions are mostly in reasonable accord with the observed 1^{-+} resonance $\pi_1(1600)$ seen by several collaborations as shown in Table 5. This is especially true for the controversial $\rho\pi$ channel which is forbidden by the gluonic excitation models (the "$L + S$" selection rule [10]).

In the constituent glue model, the nonvanishing width comes from the QE-hybrid mode ($l_g = 0$ whith P-wave $q\bar{q}$, Eq. (13)) decaying preferably into two S-wave mesons, i.e.,

$$\Gamma_{1^{-+} \to \rho\pi} \simeq 0.28^{-0.14}_{-0.09}\ \text{GeV}. \quad (26)$$

5. Conclusion

To conclude, we note that despite the imperfections of the model, the results obtained are encouraging and describe quite well the observed properties of the resonance $\pi_1(1600)$, supporting the fact that this resonance is a hybrid meson with the internal structure suggested by the generalized Quark Model with Constituent Gluon, i.e., a pair of quark-antiquark with a massive constituent gluon: $m_g \simeq 0.5 \pm 0.1$ GeV. However, this approximate model needs to be improved by considering more relativistic effects especially for the decay model. On the other hand, it would be advisable to seriously review the hypothesis that hybrids are exclusively build by excited gluonic fields.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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