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Numerical Modeling of Water Table Fluctuation in Unconfined Sloping Aquifer in Response to Multiple Localized Recharge

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Authors' contributions

This work was carried out in collaboration among all authors. Author SS designed the study, performed the literature survey, wrote the protocol and wrote the first draft of the manuscript. Author RKB managed the analysis of the study and author BS helped in the preparation of the manuscript. All authors read and approved the final manuscript.

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ABSTRACT

Numerical modeling for the variations of water table fluctuation in response to subsurface seepage and downwards recharge is an important aspect in the estimation of surface-groundwater interaction. In this work, a numerical model is developed for the approximation of water table variation in an unconfined sloping aquifer subjected to the multiple localized recharge and seepage from the adjacent water body. The Boussinesq equation characterizing the flow of groundwater in unconfined sloping porous media is solved numerically using Du Fort Frankel finite difference method. The application of the result is demonstrated with illustrative examples using varying aquifer parameters. The results indicated that the water table form groundwater mound beneath recharge basins due to continuous recharge.

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1. INTRODUCTION

Quantification of water table fluctuation due to surface-groundwater interaction is an emerging research area. One of the most powerful tools in the characterization of water flow through porous media is computational numerical models. For this, both hydrological parameters and boundary conditions need to be continuously updated considering the dynamic behaviour of hydrological conditions.

Groundwater flow in the saturated zone is approximated by a nonlinear parabolic partial differential equation, known as the Boussinesq equation. The nonlinear equation of Boussinesq is a foundational approach for studying groundwater flow through an unconfined and confined aquifer. The evolution and decay of groundwater mound due to time-varying recharge was modeled by Hantush [1] and Marino [2]. Rao and Sharma [3] presented an analytical solution of a two-dimensional linearized Boussinesq equation to predict the evolution of groundwater mounds due to constant recharge from a rectangular basin. Zomorodi [4] presented a numerical model to evaluate the mounding of groundwater table by incorporating the effect of the unsaturated zone on recharge rate. A numerical model for modeling transient stream aquifer interactions in an alluvial valley aquifer was presented by Serrano and Workman [5]. Parlange et al. [6] developed a numerical model based on the one-dimensional Boussinesq equation for a horizontal unconfined aquifer using finite element method Butler et al. [7] shows the effect of leakage and partial penetration in his work using numerical simulation technique. Groundwater table variation in a semi-infinite unconfined aquifer described the fluctuations of the water table Rai and Manglik [8]. Singh [9] in his work gave an analytical approximation to calculate the groundwater mound due to artificial recharge for an unconfined aquifer and verified the existing solutions. A numerical solution of the Boussinesq equation using a fully explicit predictor-corrector method was developed by Bansal et al. [10]. The results obtained in them indicate a considerable dependence of the aquifer's water head on the bed slopes and stream-stage variations. A boundary-value problem for movement of the critical point of disconnection is established by Wang Wang, Zhenxue Dai et al. [11] for an analytical solution of the inverted water table movement beneath the stream. Lande et al. [12] estimate the solution of an unconfined aquifer lying on a leaky base with respect to multiple recharge and with drawls. Zlotnik et al. [13] in their work gave analytical results for a transient and steady state groundwater mound in a sloping aquifer using MAR technique. Shaikh et al. [14] solved the nonlinear Boussinesq equation analytically as well as numerically to examine the effect of tidal oscillations on water table height. A closed form solution is obtained by using Fourier Cosine transform in an unconfined aquifer under seepage and multiple recharge in order to calculate the mound height and cone depression Lande et al. [15].A complete analytical solution to determining the effect of any varying rainfall recharge rates on groundwater flow in an unconfined sloping aquifer by Wu and Hsieh [16]. All of these papers show the flow simulation models which help in determining the direction of groundwater flow, distribution of hydraulic heads, and flow magnitude.

In this paper, the model consists of an unconfined aquifer resting on a downward sloping bed that is continuously in contact with an adjacent stream with a constant water level. Moreover, the aquifer also receives a downward recharge at a constant rate. The mathematical model is developed by dividing the aquifer into four-zone systems with each zone has different boundary conditions at hypothetical interfaces. The water table distinctions due to seepage of governed nonlinear Boussinesq equation are calculated using DFF numerical scheme and the results of the numerical solution developed shows the combined effects of the bed slope, recharge rate parameter on the profiles of groundwater mound.

2. DEVELOPMENT OF THE MATHEMATICAL MODEL

The theoretical model considered here consists of an unconfined sloping aquifer of semi-infinite extent. As shown in Fig. 1, the aquifer is hydrologically contacted to a water body of constant water level h_0 at its left end and is replenished constantly through two recharge basins located in the aquifer's domain. The first basin extends from $x = A_1$ to $x = A_2$ while the second recharge extends from $x = A_3$ to $x = A_4$. The flow of groundwater over sloping bed is governed by nonlinear Boussinesq equation. [17]

$$\frac{\partial}{\partial x}\left(h\frac{\partial h}{\partial x}\right)-\tan\beta\,\frac{\partial h}{\partial x}+\frac{N(x,t)}{K\cos^2\beta}=\frac{S}{K\cos^2\beta}\frac{\partial h}{\partial t} \tag{1}$$

Where

$$N(x,t)=\begin{cases}0 & 0<x\le A_1\\ N_1 & A_1<x\le A_2\\ 0 & A_2<x\le A_3\\ N_2 & A_3<x\le A_4\\ 0 & otherwise\end{cases} \tag{2}$$

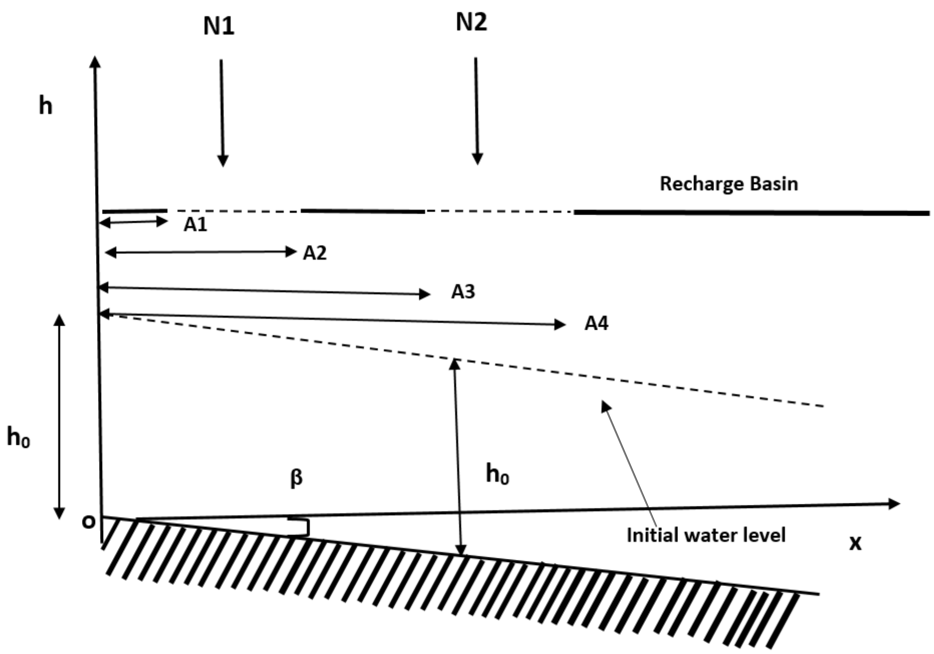

Fig. 1. Definition sketch of the unconfined aquifer receiving localized vertical recharge

The expression for the water head in an unconfined sloping aquifer under constant localized recharge can be made a special case of the main results by setting different boundary values for recharge *N (x, t)* equation (2). To solve the nonlinear Boussinesq equation (1), Du Fort Frankel (DFF) scheme, a second order explicit numerical method based on the finite difference method, is applied. This scheme is absolutely convergent and often used for the numerical solution of a parabolic partial differential equation. Implementation of the scheme proceeds in two steps, namely $j \ge 2$ and $j = 1$ where n indicates mesh number along the time axis.

Rewriting equation (1) as

$$\left\{h\frac{\partial^2 h}{\partial x^2}+\left(\frac{\partial h}{\partial x}\right)^2\right\}-\tan\beta\left(\frac{\partial h}{\partial x}\right)+\frac{N(x,t)}{K\cos^2\beta}=B_1\left(\frac{\partial h}{\partial t}\right) \tag{3}$$

where $B_1 = S/(K\cos^2\beta)$. For the discretization of the governing equation (3) apply central difference for both time and space variables, yielding

$$B_1\left(\frac{h_{i,j+1}-h_{i,j-1}}{2\Delta t}\right)=\left\{\frac{h_{i,j}\left(h_{i+1,j}-2h_{i,j}+h_{i-1,j}\right)}{(\Delta x)^2}+\frac{\left(h_{i+1,j}-h_{i-1,j}\right)^2}{(2\Delta x)^2}\right\}-\tan\beta\left(\frac{h_{i+1,j}-h_{i-1,j}}{2\Delta x}\right)+\left(\frac{N}{K\cos^2\beta}\right) \tag{4}$$

where $h_{i,j+1}$ is the water head height at the current time step. Δx and Δt denote mesh sizes along the x and t-axis respectively. Using $h_{i,j} = (h_{i,j-1} + h_{i,j+1})/2$ in the above equation and simplifying it, one gets

$$h_{i,j+1}\left(\frac{1}{2\Delta t}+\frac{h_{i,j}}{A_1(\Delta x)^2}\right)=\frac{h_{i,j-1}}{2\Delta t}+\frac{2\Delta t}{B_1}\left[\left(\frac{h_{i+1,j}-h_{i-1,j}}{2\Delta x}\right)^2-\tan\beta\left(\frac{h_{i+1,j}-h_{i-1,j}}{2\Delta x}\right)+\frac{h_{i,j}\left(h_{i+1,j}-h_{i,j-1}+h_{i-1,j}\right)}{(\Delta x)^2}\right]+(N)\frac{\Delta t}{S} \tag{5}$$

which leads to

$$h_{i,j+1}=\frac{1}{\left(\frac{1}{2\Delta t}+\frac{h_{i,j}}{A_1(\Delta x)^2}\right)}\left[\frac{h_{i,j-1}}{2\Delta t}+\frac{2\Delta t}{B_1}\left\{\left(\frac{h_{i+1,j}-h_{i-1,j}}{2\Delta x}\right)^2-\tan\beta\left(\frac{h_{i+1,j}-h_{i-1,j}}{2\Delta x}\right)+\frac{h_{i,j}\left(h_{i+1,j}-h_{i,j-1}+h_{i-1,j}\right)}{(\Delta x)^2}\right\}+(N)\frac{\Delta t}{S}\right] \tag{6}$$

Equation (6) is used for the determination of water head $h_{i,j+1}$ for $j \geq 2$. To get the value of $h_{i,j+1}$ at $j = 1$, apply forward difference discretization to temporal variables and the central difference to space variables in the equation. The resulting equation becomes

$$B_1\left(\frac{h_{i,j+1}-h_{i,j}}{\Delta t}\right)=\left\{\frac{h_{i,j}\left(h_{i+1,j}-2h_{i,j}+h_{i-1,j}\right)}{(\Delta x)^2}+\frac{\left(h_{i+1,j}-h_{i-1,j}\right)^2}{(2\Delta x)^2}\right\}-\tan\beta\left(\frac{h_{i+1,j}-h_{i-1,j}}{2\Delta x}\right)+\left(\frac{N}{K\cos^2\beta}\right) \tag{7}$$

Setting $j = 1$ in the above equation, one gets

$$h_{i,2}=h_{i,1}+\frac{\Delta t}{B_1}\left\{\frac{h_{i,1}\left(h_{i+1,1}-2h_{i,1}+h_{i-1,1}\right)}{(\Delta x)^2}+\frac{\left(h_{i+1,1}-h_{i-1,1}\right)^2}{(2\Delta x)^2}\right\}-\tan\beta\left(\frac{h_{i+1,1}-h_{i-1,1}}{2\Delta x}\right)+\left(\frac{N}{S}\right)\Delta t \tag{8}$$

The discretization of boundary conditions is given as

$$h_{i,1} = h_0$$
(9a)

$$h_{1,j+1} = h_0 \quad (9b)$$

$$h_{i,j+1} = h_0 \quad (9c)$$

The truncation error of the DFF scheme in the present case of the parabolic equation is $O(\Delta x^2) + O(\Delta t^2) + O\{(\Delta t/\Delta x)^2\}$. Keeping the convergence requirement in mind, we choose $\Delta t = 0.001$ and $\Delta x = 0.1$.

3. DISCUSSION OF RESULTS

Given model is used to solve the problems dealing with the water seepage through the adjacent stream and localized recharge and study the changes in the water table level. The values of the water table head are obtained using DFF Fortran programming by considering various values of sloping angle and aquifer parameters as h_0=5.0, K=2.5, S=0.2. Water head height at the end of time t=1,8, and 12 days in a sloping aquifer with sloping angle β = 3^0, 6^0,10^0 is shown in Figs. 2(a)- 2(c) under the local recharge rate of N_1=2 mm/h N_2=4 mm/h. The water head height in 3 deg downward sloping aquifer under the same local recharge rate but considering the four different localized regions as x=150m, 200 m, 600 m, 650 m is presented in Figs. 2(a)-2(c), it is observed that with time advancements from t=1 to t=12 days under localized recharge at different regions creates a mound which evolves with time.

The role of sloping angle β in the evolution and stabilization of groundwater mound is visible in all the above three Figs. 2(a)-2(c). It is observed from these Figures that the mound height is increased as time progresses. It is also detected from these Figures that the downward bed slope provides the natural gradient to the recharge water to flow from higher values of x.

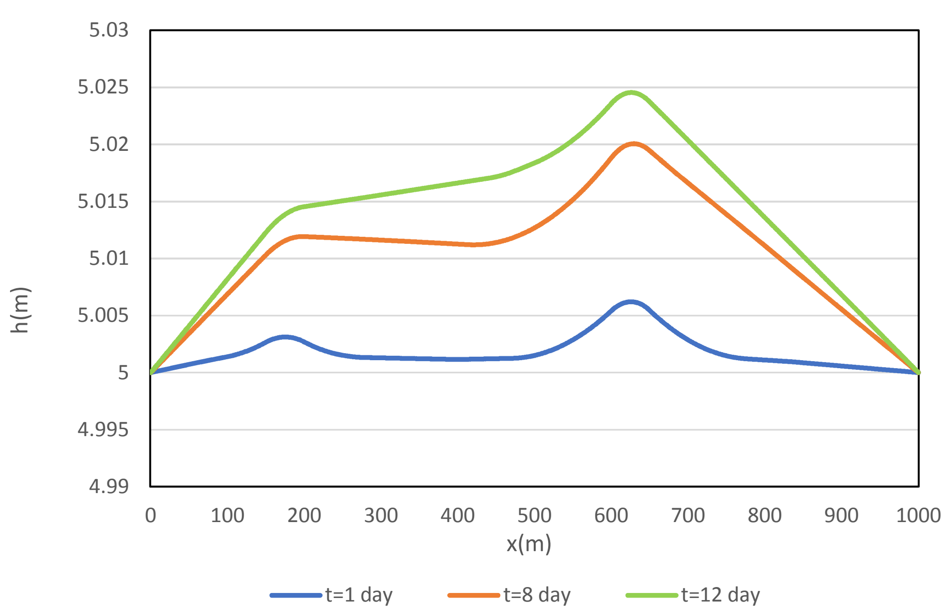

Fig. 2(a). Water head distribution at sloping angle β=3^0

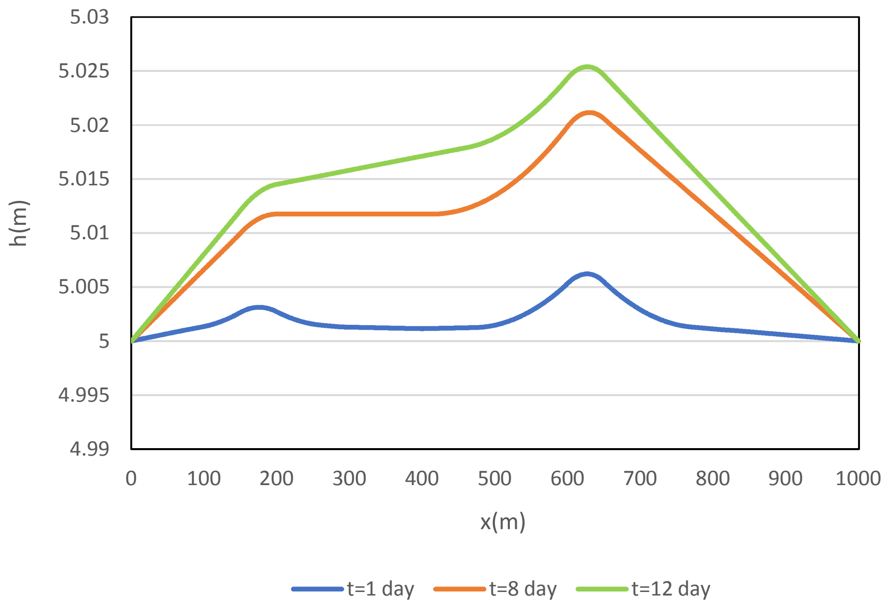

Fig. 2(b). Water head distribution at sloping angle β = 6^0

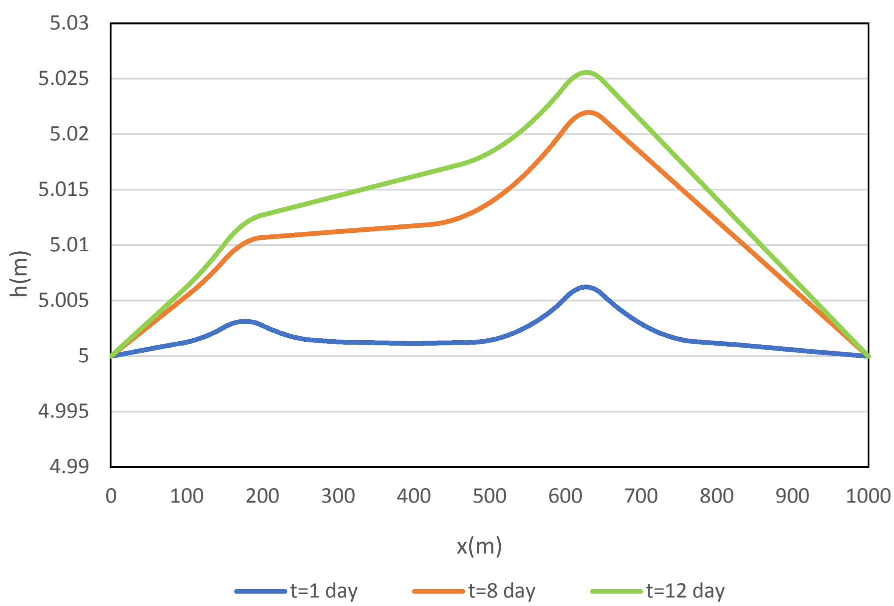

Fig. 2(c). Water head distribution at sloping angle β=10^0

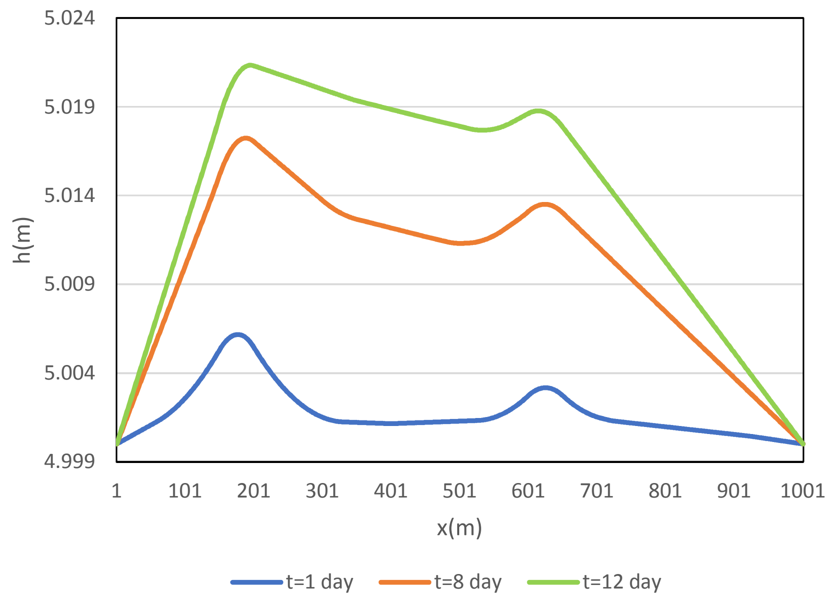

Fig. 3(a). Water head distribution at sloping angle β=3^0

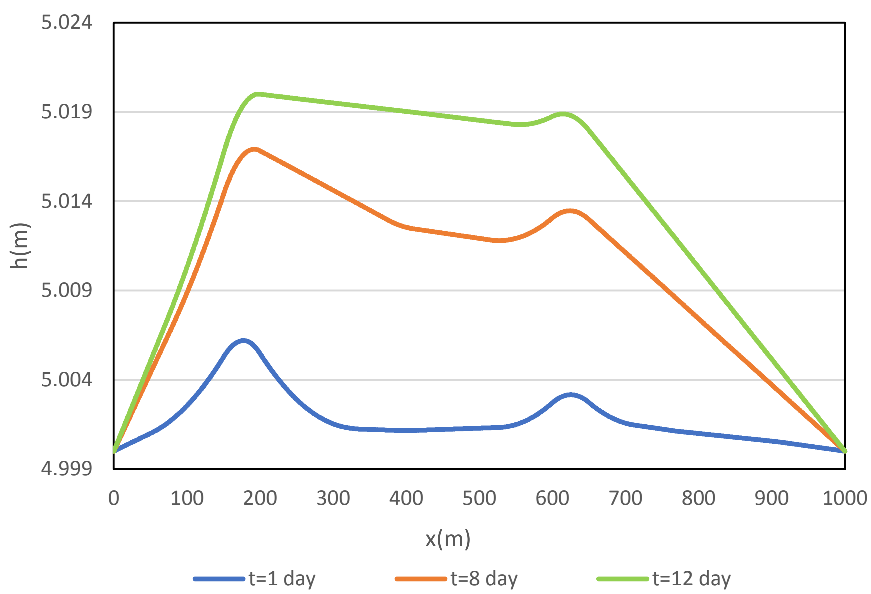

Fig. 3(b). Water head distribution at sloping angle β=6^0

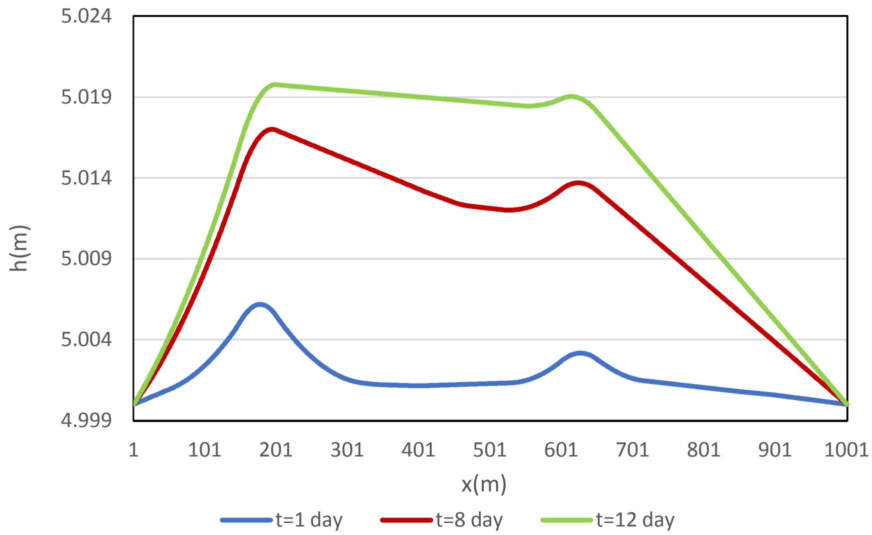

Fig. 3(c). Water head distribution at sloping angle β=10⁰

Another change observed in water head profiles is shown in Figs. 3(a)-3(c), here sloping angle β=3,6,10 deg is considered with recharge values of *N1*=4 mm/h, *N2*=2 mm/h. The effect of recharge values on the variation of mound height is clearly evident. It is observed from these Figures that if we interchange the recharge values of *N1* and *N2,* the mound height follows the reverse pattern too.

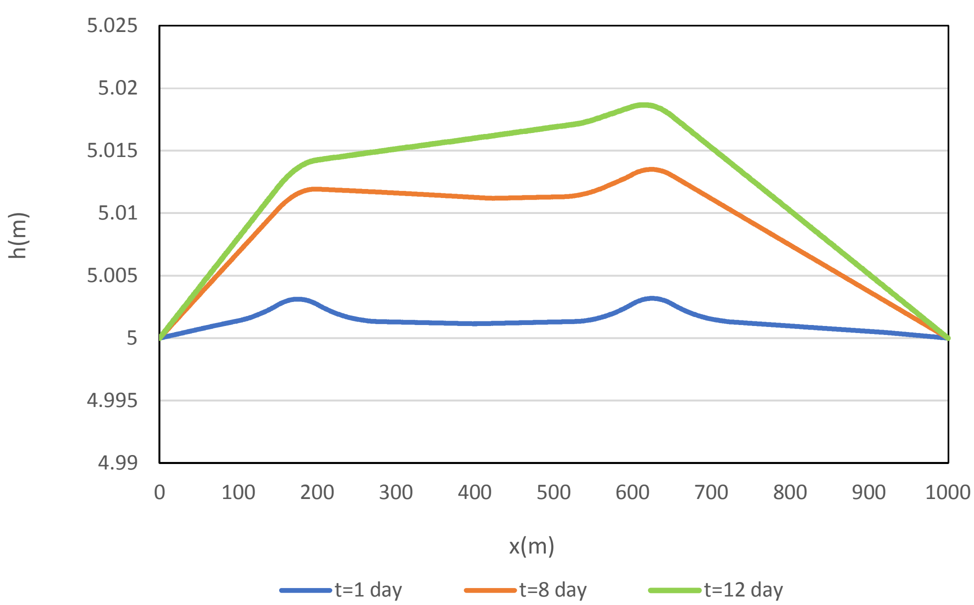

Fig. 4(a). Water head distribution for *N1=N2* at sloping angle β=3^0

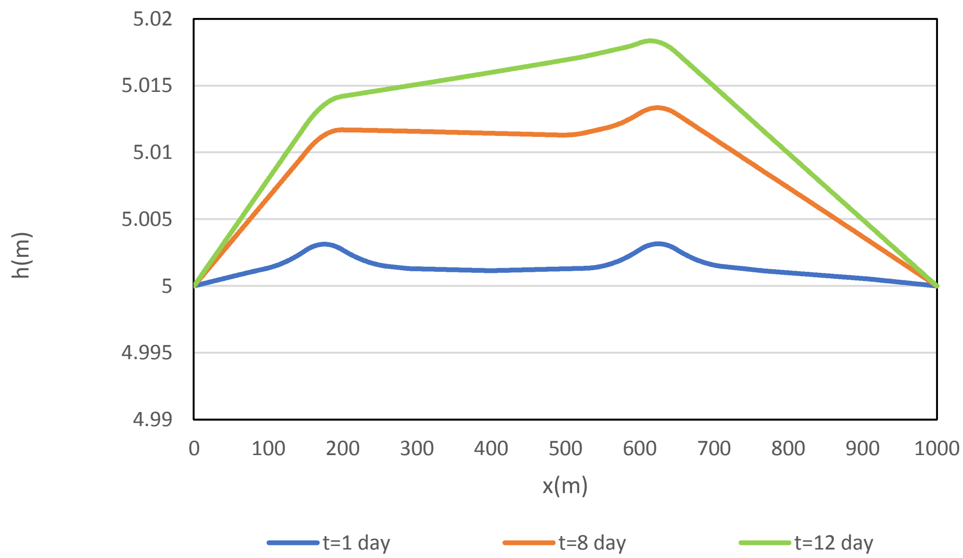

Fig. 4(b). Water head distribution for *N1=N2* at sloping angle β=6^0

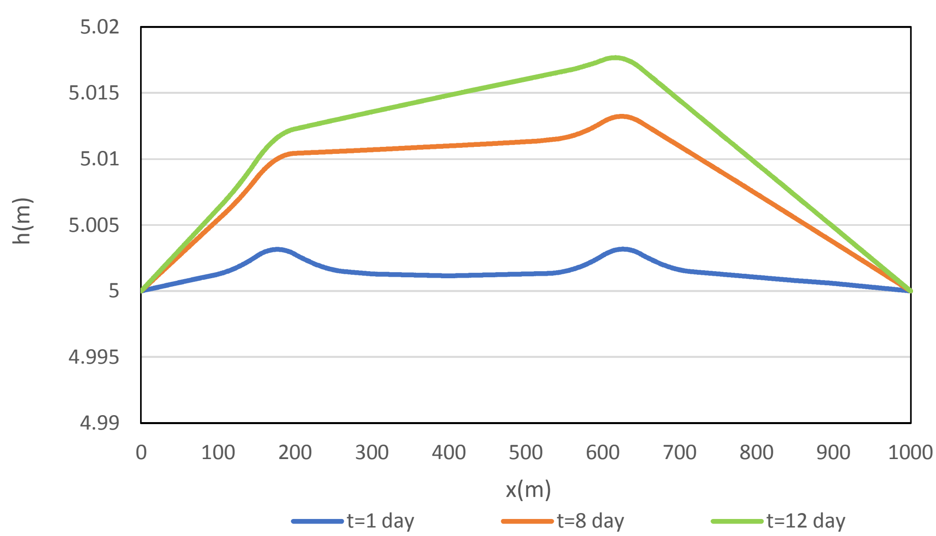

Fig. 4(c). Water head distribution for *N1=N2* at sloping angle β=10^0

Another remarkable change observed in Figs. 4(a)-4(c), when recharge values are considered as same. Here on considering *N1*= *N2*=2 mm/h with sloping angle β=3, 6, 10 deg, the changes in the mound height is clearly observed. It is obvious from these figures that when recharge values are same, the mound height settles at considerable low height as compared to mound height when we considered distinct values of recharge. The elevation of the mound is strongly affected by the depression angle. Mounding is limited in time and space and the peak of the mound moves over time. Water table mound accounts for the recharge sources at the land surface with respect to the aquifer base depression angle. The dynamic behaviour of water head profiles under the effect of spatial coordinates of recharge basin is observed in above all graphs. As bed slope increases attainting the steady state values of profile head decreases. For a longer period of time the water head profiles in a sloping aquifer become parallel to the bed.

4. CONCLUSIONS

The nonlinear unconfined flow of equation in a sloping aquifer under constant recharge is solved by a numerical scheme and dynamic profiles of the water table are plotted for different sets of aquifer parameters with a variety of sloping angles. The results obtained from numerical scheme shows the role of different aquifer parameter in the formation of groundwater mound beneath the recharge basin.

The results exclaimed that peak mound height is greatly affected by the aquifer base slope and the length of time it takes for a mound to develop. The model results can be used to validate numerical models using distinct recharge parameters and provide fieldwork guidelines.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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