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Stochastic Dominance, Dynamic Programming and Bayes Filtering: Applications in NPV and NPVaR

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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Short Research Article

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ABSTRACT

Applications of three computational methods in the evaluation of NPV and NPV at risk (NPVaR) of a project are studied. These methods are stochastic dominance, dynamic programming and Bayes filtering. First, the definition of NPVaR is related to the stochastic dominance as a maximization problem, then the dynamic programming method is used to solve the maximization in the presence of an investor belief parameter. To enter this parameter to the problem, the Bayes filter method is applied. Literatures is reviewed about these three methods. The sensitivity analysis of the results to the different utility functions as well as to the continuous compounding methods are also studied. A real data set is presented. Finally, a conclusion section is also given.

Keywords: Bayes filter; cash flow; dynamic programming; NPVaR; sensitivity analysis; stochastic dominance; utility function.

1. INTRODUCTION

The current paper considers applications of three famous methods including stochastic dominance, dynamic programming and Bayes filtering in evaluation of the utility obtained by an investor from a project. The utility function defined as the sum of discounted present value of the cash

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flows of the project. This utility is used for comparing the project to the best available alternatives. The paper has three parts related to the applications of the three methods. First, the definition of NPVaR (introduced by [1]) is related to the stochastic dominance, see [2,3]. Indeed, assuming (identical distribution and) independence, the expected value of this sum equals to the sum of expected values. Thus, a project dominates the other if its NPVaR is at least better than the NPVaR of alternative options. In this case, the project is considered superior. The second method is dynamic programming, see [4,5]. In this paper, the dynamic programming is defined as a maximization problem involving the utility function. To this end, the investor belief regarding the superiority of a given cash flow (an intuitive idea) is also considered. Thus, the dynamic programming considers this belief as a parameter of maximization process. The dynamic programming is solved for the expected value and variance of NPV assuming these follow a normal distribution. Finally, the third method, namely the Bayes filtering, is used about prediction using to the case of NPV distribution (see [6,7]). In this paper, three parts are considered for three methods. Assumptions and conditions under which the model will work are clarified. Also, the applications of each method are given. Also, Finally, this study also provides a detailed description of the circumstances under which this case of the model will work.

The rest of paper is organized as follows. In the next section, three methods (parts) are presented and their relations to NPV and NPVaR are studied. A literature review on the three methods is provided in section 3. The sensitivity analysis is studied in section 4. A real data set is surveyed in section 5. Conclusions are given in section 6.

2. LITERATURE REVIEW

The above three mentioned methods have many financial applications. In this section, some of these applications are given. For the sake of brevity, we only provide the newest references. First, the stochastic dominance is considered. [8] considered the application of stochastic dominance technique for constructing a portfolio in which its omega index is optimized. [9] applied the stochastic dominance to clarify the mispricing in index-based options. [8] used the stochastic dominance technique to identify the properties of a portfolio which has optimized its omega index. The second concept is the dynamic programming. This is a recursive computationally method for maximization of a specified function. [10] studied the application of dynamic programming of merchant operation of the commodity and the energy merchant. [11] used the dynamic programming to construct the optimal strategies for achieving an investment portfolio. [12] solved the curses of dimensionality problems with approximate dynamic programming. [13] applied the approximate linear programming for average of cost of networks. The third method is filtering. This is a tool for removing unwanted component like noises, disturbances or errors of model. Three main tasks of a filter are smoothing, filtering and prediction. There are many types of filters say low (high)-pass filters, moving average filters, Kalman filters, particle filter and Bayes filter. In this paper, the Bayes filter is focused upon. [14] applied the filtering technique for stochastic volatility time series. [15] proposed the application of particle filtering in finance and economics. In the next section, these methods are described.

3. THREE METHODS

In this section, three methods are given and some propositions are given to clarify the applications of the three methods in NPV and NPVaR analysis of a project. To this end, consider a cash flow $\{F_i\}_{i=1}^{\infty} = (F_1, F_2, \ldots)$ of a certain investment project, the NPV, which is the z-transform of project (see [16]), may be arranged as the utility function achieved by that project defined by

$$NPV_{F_i} = U(F_1, F_2, \ldots) = \sum_{i=1}^{\infty} \frac{{F_i}^a}{(1+r)^i}. \tag{1}$$

In formula (1) a is a suitable positive number and r is the interest rate. Ye and Tiong [1] introduced the concept of NPVaR for financed infrastructure projects. A main question is how is it possible to incorporate this concept in evaluation of projects? The stochastic dominance definition is applied to this end and the question is formulated as a maximization problem. Then, the dynamic programming method is applied to solve the maximization problem. The belief of investor is included to this problem using the Bayes filter method. The three parts about the applications of three methods are presented-as follows.

3.1 Stochastic Dominance

Suppose that $\{F_i\}_{i=1}^{\infty}$ and $\{G_i\}_{i=1}^{\infty}$ are two different uncertain projects, then project $\{F_i\}_{i=1}^{\infty}$ dominates the project $\{G_i\}_{i=1}^{\infty}$ if $E\left(NPV_{F_i}\right) \geq E\left(NPV_{G_i}\right)$ which is equivalent to stochastic dominance criterion (see [17]). Let F and G be the distribution functions of NPV_{F_i} and $\mathrm{NPV}_{\mathrm{G_i}}$, respectively. One can notice that the necessary condition and also the sufficient condition which are necessary to guarantee the stochastic dominance of F with respect to G, are $F(a_0) \geq G(a_0)$ for all a_0, and $F(a_{00}) \geq G(a_{00})$ for some a_{00}, see [16]. The parameters a_0 and a_{00} are the NPVaR's, for some certain significance levels $1-\alpha_0$ and $1-\alpha_{00}$. [1] introduced the concept of NPVaR for financed infrastructure projects. The following proposition summarizes the above discussion.

Proposition 1 (Relationship with stochastic dominance): The project $\{F_i\}_{i=1}^{\infty}$ dominates the project $\{G_i\}_{i=1}^{\infty}$ if for every significance level $1-\alpha_0$, then

$$NPVaR_{\mathrm{F},1-\alpha_0} \geq NPVaR_{\mathrm{G},1-\alpha_0},$$

and for some $1-\alpha_{00}$, then

$$NPVaR_{\mathrm{F},1-\alpha_{00}} > NPVaR_{\mathrm{G},1-\alpha_{00}} \tag{2}$$

Proof: The proof is a straightforward conclusion of stochastic dominance definition.

Hereafter, the investor belief about a specified i-th cash flow is imposed in formula (2). Let J_i denote the investor belief about a specified i-th cash flow where $J_i = 1$ if investor accepts that cash flow and $J_i = 0$ if the investor doesn't accept. Then, the formulas (1) and (2) are changed and the $E(NPV_{F_i})$ is given by formula (3) as follows:

$$E(NPV_{F_i}) = U(F_1, F_2, \dots) = E\sum_{i=1}^{\infty}\frac{J_i {F_i}^a}{(1+r)^i} \tag{3}$$

Remark 1: As a referee advised, the rationale a function that involves a random constant power to the cash flow per period should be clarified. Indeed, the answer relies on the mathematics of utility function. It is a type of utility function which is applied to evaluate a financial project. Interested readers can be refer to [16].

Remark 2: As a referee suggested, the rationale behind why the investor may or may not select a given cash flow is not provided (i.e. why is $J_i = 0$ for any 'i') is needed. The answer lies in the logic of Bayes method. Sometimes investors have prior information (historical information) about a project and its cash flows. Thus, currently, they guess the future cash flow will be gain or loss. It occurs in the project with uncertain cash flows like stocks. Sometimes, it can be part of some financial contract, like callable bonds, where in each time, there is a chance that bond is called by issuer. In all cases, the cash flow are random and the belief of investor about cash flows plays important role.

Remark 3: As a referee asked, what happens to the solution when a is a function of i, that is the function of the time period. This case is not considered in this paper. However, the whole structure of dynamic programming is not changed but the function becomes too complicated. Beside this, a strategy should be chosen for selection of a at each time period. However, as it is described, this case is not studied in the current paper.

3.2 Dynamic Programming

An important question is that which cash flows F_i do maximize $E(NPV_{F_i})$? This is the dynamic programming approach to the problem (see [18]). The following proposition gives the details. That is the formula (3) is changed to formula (4) as a dynamic programming problem.

Proposition 2 (Relationship with dynamic programming): Maximization of $E(NPV_{F_i})$ with respect to $\{F_i\}_{i=1}^{\infty}$ is equivalent to dynamic programming which is given by

$$max_{\{F_i\}_{i=1}^{\infty}} E\sum_{i=1}^{\infty}\beta^i {F_i}^a J_i,$$

Where

$$\beta = \frac{1}{1+r}. \tag{4}$$

Proof: Depending a special cash follow is selected or not, it is necessary to consider the $J_i F_i$ as cash flow. This point completes the proof.

3.3 Bayes Filtering

Formula (4) defines a hybrid structure for the problem. That is the combination of two the above mentioned methods. Next, it is seen that the $(J_i, F_i), i = 1,2, \ldots$ defines a Bayes filtering framework (see [19]).

Remark 4 (Posterior simulation): Suppose that F_i's are independent and each F_i is distributed identically and normally and F_i has mean μ_i and variance σ_i^2, $i \geq 1$. Then, for $a = 1$, given $(J_1, J_2, \ldots) = (l_1, l_2, \ldots)$, the NPV has normal distribution with mean μ_{npv} and variance σ_{npv}^2,

where

$$\mu_{npv} = \sum_{i=1}^{\infty} \frac{l_i \mu_i}{(1+r)^i} \text{ and } \sigma_{npv}^2 = \sum_{i=1}^{\infty} \frac{l_i \sigma_i^2}{(1+r)^{2i}}. \quad (5)$$

Proof: The formula (5) is obtained by applying the Bayes formula directly.

In formula (5), let $l_i = 1$ for some $i = i^*$ and zero otherwise. The posterior distribution of $J_{i^*} = 1$ given NPV is the selection probability the cash flow i^* by investor after observing the sampling information which is the NPV. The reference [20] bootstrap re-sampling method is useful to simulate the posterior distribution. The procedure is to generate some sequence of prior distribution Then, the likelihood is computed for each samples and bootstrap re-sampling probability is calculated by dividing each likelihood over the summation of all the likelihoods summation. The bootstrapped samples come from the posterior distribution. To this end, it is enough to notice that, given $l_i = 1$ for some $i = i^*$ and zero otherwise, then

$$\mu_{npv} = (l_{i^*} \mu_{i^*})/(1+r)^{i^*}, \quad \sigma_{npv}^2 = l_{i^*} \sigma_{i^*}^2/(1+r)^{2i^*}. \quad (6)$$

The rest of procedure in formula (6) is too easy. From J_{i^*} which has a Bernoulli distribution, samples are generated, then likelihood and weights are computed and posterior samples are obtained.

Remark 5 (Bayesian filtering): Here, the Bayesian filtering is proposed to obtain the probability of selection of a cash flow by investor. Note that the state variables are J_i's where they are independent.

Therefore $f(J_i | J_k, k \leq i-1) = f(J_i)$. Also, $f(NPV | J_k, k \leq i)$ is normal distribution with mean μ_{npv} and variance σ_{npv}^2,

where

$$\mu_{npv} = \sum_{i=1}^{\infty} \frac{l_i \mu_i}{(1+r)^i}, \sigma_{npv}^2 = \sum_{i=1}^{\infty} \frac{l_i \sigma_i^2}{(1+r)^{2i}} \quad (7)$$

Proof: The formula (7) is obtained by applying the Bayes formula directly. Also, notice that the square of l_i is itself.

Therefore, according to the formula (7), the Bayes filtering may be applied to simulate the posterior distribution.

4. SENSITIVITY ANALYSIS

In this section, the sensitivity analysis is studied in two ways. First, different utility functions such as the CRRA function or a log utility function are considered and checked for sensitivity across these specifications. Then, the sensitivity of the model when there is continuous compounding is studied.

4.1 Types of Utility

There are many types of utility functions. For example a negative exponential function is $u(w) = 1 - e^{-cw}$ for $c > 0$ or $u(w) = w - aw^2$ for $a > 0$ and $w \leq \frac{1}{2a}$. A useful utility function is the logarithmic utility which is a limiting case of power utility

$$u(w) = lim_{\alpha \to 0} \frac{w^\alpha - 1}{\alpha} = \ln(w).$$

Here, the dynamic programming problem is given by

$$max_{\{F_i\}_{i=1}^{\infty}} E \sum_{i=1}^{\infty} \beta^i \ln(F_i) J_i,$$

Define $g(x) = \ln(x) - x^\alpha$ for $x > 0$. The first derivative of g is $\frac{1 - \alpha x^\alpha}{x}$. It is negative if for example $\alpha, x \geq M > 1$. Thus, g is decreasing its maximum value is attained at $x = 1$ which is -1. Hence, $\ln(x) < -1 + x^\alpha < x^\alpha$. So, to make sure that both maximized value don't differ, it is necessary that F_i (obtained by power utility function) be smaller that F_i (obtained by logarithmic utility function).

Remark 6: As suggested by a referee, an important type of utility function is the constant relative risk aversion (CRRA) utility which is given by

$$u(w) = \frac{1}{1-\theta} w^{1-\theta}, \theta \neq 1, \theta > 0.$$

Here, the dynamic programming problem is given by

$$max_{\{F_i\}_{i=1}^{\infty}} E \sum_{i=1}^{\infty} \beta^i {F_i}^{1-\theta} J_i,$$

The results of dynamic programming in this case corresponds to the general problem with $a = 1 - \theta$.

4.2 Continuous Compounding

Another topic which is interested in sensitivity analysis is the effect of continuous compounding. In this case the dynamic programming is given by

$$max_{\{F_i\}_{i=1}^{\infty}} E \sum_{i=1}^{\infty} \beta^{*i} {F_i}^{a} J_i,$$

where $\beta^* = e^{-r}$. As follows, the plot of $\beta - \beta^*$ for various values of r is given in Fig. 1.

It is seen that $> \beta^*$, for all r's. Thus, to make sure that both maximized value don't differ, it is necessary that F_i (obtained by discrete compounding) be smaller that F_i (obtained by continuous compounding).

5. REAL DATA SET

Here, a real life application of this model seems to be working are illustrated. Here, the re-sampling method used is the conventional Monte Carlo approach. The data set is the price of stock of Intel corporation during 21 Jun 2016 to 8 December 2016. Its return series R is calculated and plotted in the following figure (Fig. 2). A first order autoregressive model is fitted to these returns with coefficient -0.00665. Therefore, $F_i = F_0 \prod_{j=1}^{i}(1 + R_j)$, where $R_j = \alpha R_{j-1} + \varepsilon_j$ where $\alpha = -0.00665$ and $F_0 = 35.7$. It is seen that ε_j is distributed as normal with mean zero and variance 0.000182.

The maximization problem is given by $max_{\alpha} E \sum_{i=1}^{\infty} \beta^i F_i J_i$. Here, assuming the probability of success of Bernoulli variables J_i's is $p = 0.5$ and the compounding rate is $r = 0.05$, the maximum value for α is given by $\alpha = 0.35$. This is the best ideal performance of Intel corporation. This suggests to the investor, given the current circumstances, the best time for trading the Intel stock is when the value of α is close to 0.35.

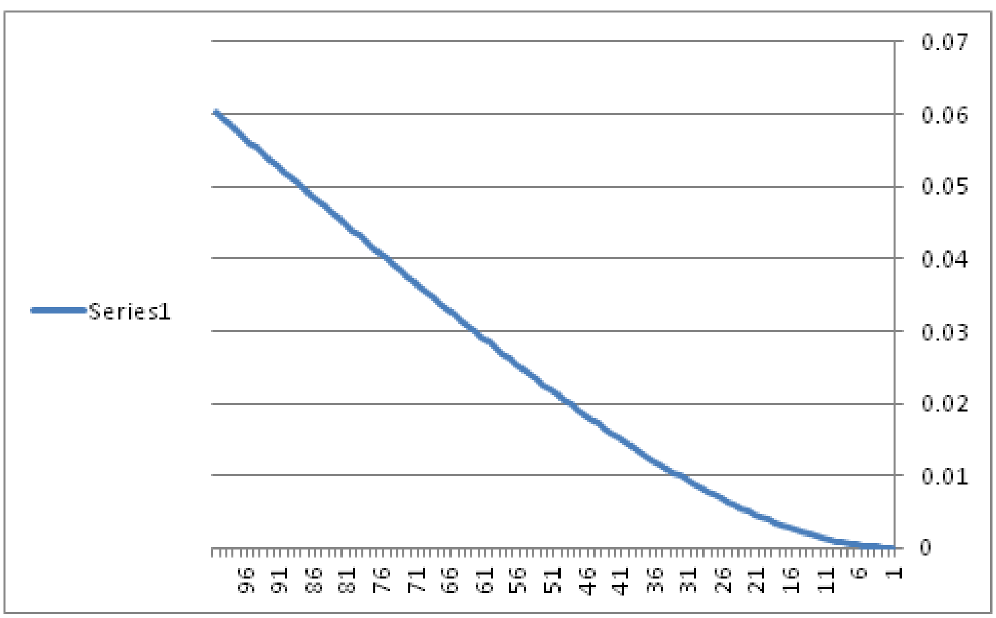

Fig. 1. The plot of $\beta - \beta^*$

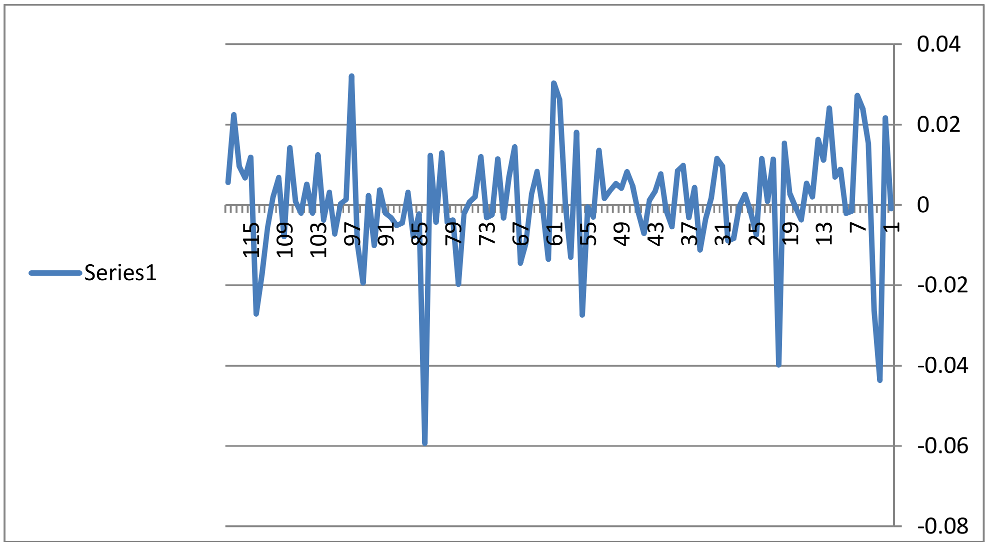

Fig. 2. Time series plot of Intel returns

6. CONCLUSIONS

The sum of discounted cash flow defined as NPV is usually used to evaluate a project. A closely related concept to the NPV is the NPVaR criterion. To evaluate performance of a project using this criterion a procedure is needed. In this note, first, it is shown that the concept of NPVaR can be represented as the stochastic dominance. Then, it is seen that the dynamic programming approach works well to solve the maximization of the stochastic dominance problem. The sensitivity analysis shows that the problem may be considered by many different types of utility functions defined on cash flows. The belief of investor on a specified cash flow itself is an important parameter which is included to the problem by defining a dummy binary variable. This modification causes to advise the use of Bayes filtering technique. Finally, this technique is used in the Intel real data set and suggests the best time for trading this stock is when the coefficient of its return in autoregressive model is close to 0.35.

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COMPETING INTERESTS

Author has declared that no competing interests exist.

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