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A Group Decision Making Method Based on Projection Method and Score Function under IVIFS Environment

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Abstract

In the paper, we will consider the interval-valued intuitionistic fuzzy multiple attribute group decision making problems. The paper presents a method to derive the weights of experts and to rank the preference order of alternatives based on projection, fuzzy entropy and score functions. Firstly, we obtain the weight of decision makers according to the projection of the individual decision on the mean decision matrix. Then, basing on the score function and fuzzy entropy we develop a practical algorithm to rank alternatives. Finally, an illustrative example is given to verify the developed method and demonstrate its practicality and effectiveness.

Keywords: Interval-valued intuitionistic fuzzy set; multiple attribute group decision making; projection method; score function; fuzzy entropy.

1 Introduction

Atanassov and Gargov extended the intuitionistic fuzzy set (IFS) to the interval- valued intuitionistic fuzzy set (IVIFS) [1-3], characterized by membership and non-membership function whose values are intervals rather than real members. Since IVIFS were proposed, a great deal of literature abounds on both theoretical research and on application research in various fields, such as engineering, economics, and management.

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In particular, IVIFS is effective in solving the decision making problems. In most multiple attribute decision making (MADM) problems, the preference over alternatives provided by decision makers is usually not sufficient for crisp membership and non-membership degree values, because things are fuzzy, uncertain and probably influenced by the subjectivity of the decision makers, or the knowledge and data about the problem domain are insufficient during the decision making process. Therefore, the preference among alternatives with uncertainty may be denoted by IFS and IVIFS for decision making problems. Besides, the current socio-economic environment is becoming more and more complex, which makes it almost impossible for a single decision maker to consider all the aspects of a problem. Generally, several decision makers are involved in the decision making, that is the group decision making problem.

Many experts and scholars have carried on research of applied IVIFSs to the multiple attribute group decision making (MAGDM) problems. Atanassov [3] obtained an intuitionistic fuzzy interpretation of MAGDM problems, in which each decision maker is asked to evaluate at least a part of the alternatives in terms of their performance with respect to each predefined attribute. They also developed a method for MAGDM problems and proposed some examples. Aiming at interval-valued intui-tionistic fuzzy MAGDM problems. Xu [4] put forward a new method by defining the ideal point and negative point of the Euclidean distances. Combined with the basic idea of TOPSIS, Ye [5] get a method of MAGDM problem with the attribute weights and expert weights given. Ye [6] proposed a MADM method based on weighted correlation coefficients and entropy weights under IVIFS environment and criteria weights for alternatives completely unknown. Bai [7] gave an improved score function for the effective ranking order of IVIFSs and proposed a MADM TOPSIS method based on an improved score function with the criteria weights known. Wei [8] presented a new method for handing MADM problems based on intuitionistic fuzzy induced geometric aggregation operators, then extended the results to interval-valued intuitionistic fuzzy MAGDM problems. By defining geometric operator and score function Park [9] gave a method of the MAGDM problem with incomplete information on attribute weights. Zhang [10] put forward two methods to determine attribute weights in MAGDM under IVIFS with incomplete attribute weight information. In the first method, the $H_{p,q}$ operator, fuzzy linear programming and extended TOPSIS were integrated to derive attribute weights. In the second method, a series of programming models based on cross-entropy were constructed and eventually aggregated into a single objective linear programming model, from which the attribute weights are obtained. Sun [14] proposed a MADM method based on fuzzy entropy and scoring function under the condition of the criterion information is completely unknown .

In all these existing approaches, the weights of the experts are the same for all the attributes. However, if the weights of experts for all the attributes are same, the evaluating results would be unreasonable, as different experts have their own knowledge and experience in reality and they are actually experts in some of the attribute and not in other attribute. Hence, the different weights of the decision makers should be assigned to different attributes in the MAGDM problems. Inspired by this idea, some experts and scholars began to solve the problem. Ye [11] proposed entropy weighted models to determine the weights of both experts and attributes from decision matrices under IFS and IVIFS environment. Ye [12] obtained weight models based on score function to determine the weights of both experts and attributes from decision matrices under IFS and IVIFS environment. Zhao [15] gave a MAGDM projection method under the IVIFS environment. We propose a projection method to derive the experts' weight in IVIFS environment. Especially, the expert whose evaluation is close to ideal decision has a large weight, while the expert whose evaluation value is far from ideal decision has a small weight. Furthermore, the preference order of alternatives can be ranked in accordance with weighted score function.

The rest of this paper is organized as followings: In section 2, we give some basic concepts. In section 3, we propose a practical algorithm to derive the weights of experts and to rank the preference order of alternatives based on projection, fuzzy entropy and score functions. Finally, an illustrative example is given to verify the developed method and demonstrate its practicality and effectiveness.

2 Preliminaries

Interval intuitionistic fuzzy set was first introduced by Atanassov and Gargov, it is characterized by an interval-valued membership degree and an interval-valued non-membership degree.

Definition 1([2])

Let a set X be fixed, an IVIFS in X is an object of the following term:

$$A = \{< x, \mu_A(x), \nu_A(x) > \mid x \in X\}, \quad (1)$$

where

$$\mu_A(x) = [\mu_A^L(x), \mu_A^U(x)] \subseteq [0,1] \text{ and } \nu_A(x) = [\nu_A^L(x), \nu_A^U(x)] \subseteq [0,1],$$
$$\mu_A^L(x) = \inf \mu_A(x),\ \mu_A^U(x) = \sup \mu_A(x),$$
$$\nu_A^L(x) = \inf \nu_A(x),\ \nu_A^L(x) = \sup \nu_A(x),$$

and

$0 \le \mu_A^U(x) + \nu_A^U(x) \le 1$, for all $x \in X$.

Let $\pi_A(x) = [\pi_A^L(x), \pi_A^U(x)]$, where

$$\pi_A^L(x) = 1 - \nu_A^U(x) - \nu_A^U(x), \quad \pi_A^U(x) = 1 - \nu_A^L(x) - \nu_A^L(x), \text{ for all } x \in X. \quad (2)$$

For convenience, we denote interval-valued intuitionistic fuzzy number (IVIFN) ([13]) by $\alpha = (\mu_\alpha, \nu_\alpha, \pi_\alpha)$, where

$$\mu_\alpha = [\mu_\alpha^L, \mu_\alpha^U] \subseteq [0,1], \nu_\alpha = [\nu_\alpha^L, \nu_\alpha^U] \subseteq [0,1], 0 \le \mu_\alpha^U + \nu_\alpha^U \le 1,$$
$$\pi_\alpha = [\pi_\alpha^L, \pi_\alpha^U] = [1 - \nu_\alpha^U - \nu_\alpha^U, \pi_\alpha^U = 1 - \nu_\alpha^L - \nu_\alpha^L].$$

Definition 2([13])

Let $X = \{x_1, x_2, ..., x_n\}$ be a finite universe of discourse, A be an IVIFS in X, then

$$|A| = \sqrt{\sum_{i=1}^{n} |\alpha_i|^2} \quad (3)$$

is called the module of A, where $\alpha_i = (\mu_{\alpha_i}, \nu_{\alpha_i}, \pi_{\alpha_i})$ is the i-th IVIFN of A, and $|\alpha_i|$ is the module of α_i, which can be denoted as follows:

$$|\alpha_i| = \sqrt{(\mu_{\alpha_i}^L)^2 + (\mu_{\alpha_i}^U)^2 + (\nu_{\alpha_i}^L)^2 + (\nu_{\alpha_i}^U)^2 + (\pi_{\alpha_i}^L)^2 + (\pi_{\alpha_i}^U)^2}. \quad (4)$$

Definition 3 ([13])

Let $X=\{x_1,x_2,...,x_n\}$ be a finite universe of discourse, A and B be two IVIFSs in X, then

$$\Pr j_B A=\frac{\sum_{i=1}^{n}(\mu_{\alpha_i}^L\mu_{\beta_i}^L+\mu_{\alpha_i}^U\mu_{\beta_i}^U+\nu_{\alpha_i}^L\nu_{\beta_i}^L+\nu_{\alpha_i}^U\nu_{\beta_i}^U+\pi_{\alpha_i}^L\pi_{\beta_i}^L+\pi_{\alpha_i}^U\pi_{\beta_i}^U)}{|B|} \tag{5}$$

is called the projection of A on B, where $\alpha_i=(\mu_{\alpha_i},\nu_{\alpha_i},\pi_{\alpha_i})$ and $\beta=(\mu_{\beta_i},\nu_{\beta_i},\pi_{\beta_i})$ are the i-th IVIFN of A and B. Obviously, the greater the value $\Pr j_B A$, the more the degree of the A approaching to B. Especially, if $n=1$, then we get the projection of IVIFN $\alpha_1=(\mu_{\alpha_1},\nu_{\alpha_1},\pi_{\alpha_1})$ on $\beta_1=(\mu_{\beta_1},\nu_{\beta_1},\pi_{\beta_1})$ as :

$$\Pr j_{\beta_1}\alpha_1=\frac{\mu_{\alpha_1}^L\mu_{\beta 1}^L+\mu_{\alpha 1}^U\mu_{\beta 1}^U+\nu_{\alpha 1}^L\nu_{\beta_1}^L+\nu_{\alpha 1}^U\nu_{\beta 1}^U+\pi_{\alpha_1}^L\pi_{\beta 1}^L+\pi_{\alpha_1}^U\pi_{\beta_1}^U}{|\beta_1|}. \tag{6}$$

Definition 4([13]) (Score function)

Let $\alpha=([a,b],[c,d])$is an interval intuitionistic fuzzy set, then define

$$S(\alpha)=(a-c+b-d)/2 \tag{7}$$

as the score function of α, $S(\alpha)\in[-1,1]$. Obviously, $S(\alpha)$ is monotonically increasing with α.

But the formula (7) is flawed. For example, when

$$\alpha_1=\{[0.4,0.5],[0.4,0.5]\},\ \alpha_2=\{[0.2,0.3],[0.2,0.3]\}$$

and $\omega\neq 0$, we have $S(\alpha_1)=S(\alpha_2)$, That is to say the score function can't compare α_1 and α_2. In order to solve this problem, we use the weighted arithmetic averaging operator to compensate:

Definition 5([13]) (Weighted arithmetic averaging operator)

Let $\alpha_j=([a_j,b_j],[c_j,d_j])\,j=1,2,...,n$ is a series of interval-valued intuitionistic fuzzy set, then we define the weighted arithmetic averaging operator under the n dimension interval valued intuitionistic fuzzy environment as follows:

$$F_\omega(\alpha_1,\alpha_2,...,\alpha_n)=\sum_{j=1}^{n}\omega_j\alpha_j=\{[1-\prod_{j=1}^{n}(1-a_j)^{\omega_j},1-\prod_{j=1}^{n}(1-b_j)^{\omega_j}],\ [\prod_{j=1}^{n}c_j^{\omega_j},\prod_{j=1}^{n}d_j^{\omega_j}]\}, \tag{8}$$

where ω_j is the weight of α_j, $\omega_j\in[0,1]$, $\sum_{j=1}^{n}\omega_j=1$.

3 Multiple Attribute Group Decision Making Problem

We consider the group decision making problem in this section. Assume that there are m alternatives $A=\{A_1,A_2,...,A_m\}$ and n decision criteria $G=\{G_1,G_2,...,G_n\}$, the evaluation value of the alternative A_i with respect to the attribute G_j is represented by the IVIFNs. Let $R^k = (r_{ij}^k)_{m\times n} = (t_{ij}^k, f_{ij}^k, \pi_{ij}^k)_{m\times n}$ be an interval-valued intuitionistic fuzzy decision matrix. $(t_{ij}^k, f_{ij}^k, \pi_{ij}^k)_{m\times n}$ is the corresponding IVIFN provided by the decision maker D_k for the alternative A_i with respect to the attribute G_j. Here, t_{ij}^k indicates the degree that the alternative A_i should satisfy the attribute G_j, f_{ij}^k indicates the degree that the alternative A_i should not satisfy the attribute G_j and π_{ij}^k indicates the degree that the alternative A_i is determined to the attribute G_j. For convenience of calculation, let

$$t^k{}_{ij} = [t_{ij}^{L(k)}, t_{ij}^{U(k)}] \subseteq [0,1],\ f^k{}_{ij} = [f_{ij}^{L(k)}, f_{ij}^{U(k)}] \subseteq [0,1],$$
$$\pi^k{}_{ij} = [\pi_{ij}^{L(k)}, \pi_{ij}^{U(k)}] \subseteq [0,1],$$

and

$$0 \le f_{ij}^{U(k)} + t_{ij}^{U(k)} \le 1,$$
$$\pi_{ij}^{L(k)} = 1 - t_{ij}^{U(k)} - f_{ij}^{U(k)}, \pi_{ij}^{U(k)} = 1 - t_{ij}^{L(k)} - f_{ij}^{L(k)}\ (i = 1,2,\cdots m;\quad j = 1,2,\cdots,n).$$

In many practical problems, the importance of each criterion decision problems require different, so we needs to know the criterion of information. But in many cases, because of the time, knowledge and lack of data or expert knowledge and many other factors, the importance of each criterion is unable to determine, and the weight is completely unknown. Ye [6] presented a model of fuzzy entropy and weighted correlation coefficient, using the model to determine the weight of each criterion value.

If we are completely unknown about criterion $G_j (j = 1,2,...,n)$, in order to determine the weight of each criterion, paper [6] determined a model of fuzzy entropy as follow:

$$\omega_j = \frac{1 - H_j}{n - \sum_{j=1}^n H_j}, \tag{9}$$

where $\omega_j \in [0,1], \sum_{j=1}^n \omega_j = 1$,

$$H_j = \frac{1}{m}\sum_{i=1}^m \{\{\sin\frac{\pi\times[1 + a_{ij} + p(b_{ij} - a_{ij}) - c_{ij} - q(d_{ij} - c_{ij})]}{4} + \sin\frac{\pi\times[1 - a_{ij} - p(b_{ij} - a_{ij}) + c_{ij} + q(d_{ij} - c_{ij})]}{4} - 1\}\times\frac{1}{\sqrt{2}-1}\} \tag{10}$$

or

$$H_j = \frac{1}{m}\sum_{i=1}^m \{\{\cos\frac{\pi\times[1 + a_{ij} + p(b_{ij} - a_{ij}) - c_{ij} - q(d_{ij} - c_{ij})]}{4} + \cos\frac{\pi\times[1 - a_{ij} - p(b_{ij} - a_{ij}) + c_{ij} + q(d_{ij} - c_{ij})]}{4} - 1\}\times\frac{1}{\sqrt{2}-1}\} \tag{11}$$

and $0 \le H_j \le 1 (j = 1,2,\cdots,n)$.

In the following, we propose a procedure for MAGDM problems with interval valued intuitionistic fuzzy information by application of projection method. The procedure involves the following steps:

Algorithm:

The decision makers evaluate the alternative with respect to the attributes to form the interval valued intuitionistic fuzzy decision matrices

$$R^k = (r_{ij}^k)_{m\times n} = (t_{ij}^k, f_{ij}^k, \pi_{ij}^k)_{m\times n};$$

Step 1: Define the mean of these evaluation values as:

$$R^* = (r_{ij}^*)_{m\times n} = (t_{ij}^*, f_{ij}^*, \pi_{ij}^*)_{m\times n} = ([t_{ij}^{L*}, t_{ij}^{U*}],[t_{ij}^{L*}, t_{ij}^{U*}][t_{ij}^{L*}, t_{ij}^{U*}])_{m\times n},$$

where

$$\begin{aligned} &t_{ij}^{L*} = \frac{1}{t}\sum_{k=1}^{t} t_{ij}^{L(k)},\quad t_{ij}^{U} = \frac{1}{t}\sum_{k=1}^{t} t_{ij}^{U(k)},\quad f_{ij}^{L*} = \frac{1}{t}\sum_{k=1}^{t} f_{ij}^{L(k)},\\ &f_{ij}^{U*} = \frac{1}{t}\sum_{k=1}^{t} f_{ij}^{U(k)},\quad \pi_{ij}^{L*} = \frac{1}{t}\sum_{k=1}^{t} \pi_{ij}^{L(k)},\quad \pi_{ij}^{U*} = \frac{1}{t}\sum_{k=1}^{t} f_{ij}^{U(k)},\\ &(i=1,2,\cdots m;\quad j=1,2,\cdots,n). \end{aligned} \tag{12}$$

Step 2: Calculate the projection of each evaluation r_{ij}^k on the mean value r_{ij}^* by (6) as Follows:

$$\Pr j_{r_{ij}^*} r_{ij}^{(k)} = \frac{t_{ij}^{L(k)} t_{ij}^{L*} + t_{ij}^{U(k)} t_{ij}^{U*} + f_{ij}^{L(k)} f_{ij}^{L*} + f_{ij}^{U(k)} f_{ij}^{U*} + \pi_{ij}^{L(k)} \pi_{ij}^{L*} + \pi_{ij}^{U(k)} \pi_{ij}^{U*}}{\left| r_{ij}^* \right|}. \tag{13}$$

Therefore, the weight for r_{ij}^k can be defined as follows:

$$\lambda_{ij}^{(k)} = \frac{\Pr j_{r_{ij}^*} r_{ij}^{(k)}}{\sum_{k=1}^{t} \Pr j_{r_{ij}^*} r_{ij}^{(k)}}, k=1,2,\ldots,t, i=1,2,...,m, j=1,2,...,n. \tag{14}$$

Step 3: After obtaining these weights, we can aggregate the evaluation values provided by different experts through

$$r_{ij} = \lambda_{ij}^{(1)} r_{ij}^{(1)} + \lambda_{ij}^{(2)} r_{ij}^{(2)} + \ldots + \lambda_{ij}^{(t)} r_{ij}^{(t)}, \tag{15}$$

where $r_{ij} = ([t_{ij}^L, t_{ij}^U],[f_{ij}^L, f_{ij}^U],[\pi_{ij}^L, \pi_{ij}^U])$, thus we can obtain the decision matrix $R = (r_{ij})_{m\times n}$.

After the steps above, the group decision making problem can be reduced to a decision making problem. Then follow the algorithm of paper [10] to solve the decision making problem.

Firstly, following from (9) and (10) or (11), the weight of each criterion ω_j is determined.

Secondly, the comprehensive evaluation $z_i(\omega)$ of each alternative $A_i (i=1,...m)$ can be given by (8):

$$z_i(\omega)=([a_i,b_i],\ [c_i,d_i])=([1-\prod_{j=1}^{n}(1-a_j)^{w_j},\ 1-\prod_{j=1}^{n}(1-b_j)^{w_j}],\ [\prod_{j=1}^{n}c_j^{\ w_j},\ \prod_{j=1}^{n}d_j^{\ w_j}]) \tag{16}$$

Then, we can obtain the weighted score function value of the comprehensive evaluation $z_i(\omega)$ by (7):

$$S(z_i)=(a_i-c_i+b_i-d_i)/2\,. \tag{17}$$

Therefore，according to the score function value all the alternatives can be ranked and the best alternative can be selected.

4 Illustrative Example

Now，we discuss a problem concerning with a manufacturing company searching the best global supplier for one of its most criteria parts used in assembling process. The attributes which are considered here in selection of four potential global suppliers, i.e., the set of alternatives is $A=\{A_1,A_2,A_3,A_4\}$ are (1) G_1: overall cost of the product; (2) G_2 :the quality of the product; (3) G_3 : supplier's profile; An expert group is formed which consists of four experts from each strategic decision area. By statistical methods, the expert $D_k(k=1,2,3,4)$ evaluates the characteristics of the potential global supplier $A_i(i=1,2,3,4)$ with respect to the attribute $G_j(j=1,2,3)$ by interval-valued intuitionistic fuzzy numbers. Thus the four decision matrices can be obtained in Table 1.

Table 1. All experts' interval-valued decision making matrix

Decision	Alternative	Attribute G_1	Attribute G_2	Attribute G_3
D_1	A_1	$<[0.70,0.80],[0.10,0.15]>$	$<[0.60,0.65],[0.25,0.30]>$	$<[0.70,0.80],[0.15,0.15]>$
	A_2	$<[0.70,0.75],[0.15,0.20]>$	$<[0.65,0.7],[0.25,0.30]>$	$<[0.70,0.75],[0.2,0.25]>$
	A_3	$<[0.80,0.85],[0.1,0.15]>$	$<[0.70,0.70],[0.20,0.25]>$	$<[0.70,0.80],[0.10,0.15]>$
	A_4	$<[0.60,0.65],[0.25,0.30]>$	$<[0.70,0.80],[0.15,0.20]>$	$<[0.70,0.75],[0.15,0.20]>$
D_2	A_1	$<[0.70,0.75],[0.20,0.25]>$	$<[0.65,0.70],[0.25,0.30]>$	$<[0.80,0.85],[0.10,0.15]>$
	A_2	$<[0.65,0.70],[0.25,0.30]>$	$<[0.70,0.75],[0.25,0.25]>$	$<[0.70,0.75],[0.15,0.20]>$
	A_3	$<[0.75,0.80],[0.15,0.20]>$	$<[0.75,0.80],[0.15,0.20]>$	$<[0.75,0.80],[0.15,0.20]>$
	A_4	$<[0.75,0.80],[0.20,0.20]>$	$<[0.55,0.60],[0.35,0.40]>$	$<[0.60,0.70],[0.20,0.25]>$
D_3	A_1	$<[0.80,0.85],[0.10,0.12]>$	$<[0.60,0.65],[0.30,0.35]>$	$<[0.80,0.85],[0.10,0.15]>$
	A_2	$<[0.80,0.85],[0.10,0.15]>$	$<[0.70,0.75],[0.20,0.25]>$	$<[0.80,0.83],[0.05,0.07]>$
	A_3	$<[0.65,0.70],[0.25,0.27]>$	$<[0.85,0.90],[0.05,0.10]>$	$<[0.62,0.65],[0.28,0.30]>$
	A_4	$<[0.80,0.85],[0.10,0.15]>$	$<[0.75,0.79],[0.12,0.16]>$	$<[0.65,0.70],[0.25,0.25]>$
D_4	A_1	$<[0.70,0.75],[0.20,0.25]>$	$<[0.75,0.80],[0.10,0.15]>$	$<[0.68,0.70],[0.22,0.25]>$
	A_2	$<[0.59,0.65],[0.21,0.25]>$	$<[0.78,0.80],[0.12,0.15]>$	$<[0.80,0.82],[0.10,0.13]>$
	A_3	$<[0.56,0.60],[0.24,0.25]>$	$<[0.80,0.85],[0.10,0.15]>$	$<[0.60,0.65],[0.23,0.30]>$
	A_4	$<[0.80,0.80],[0.10,0.18]>$	$<[0.70,0.75],[0.15,0.20]>$	$<[0.67,0.70],[0.15,0.20]>$

Thus, we can utilize the proposed method to obtain the most desirable alternative. We utilize algorithm proposed in paper to determine the expert weights.

We first derive the mean of these evaluation values $R^*=(r_{ij}^*)_{4\times 3}$ by formula (12) (see Table 2).

After that, we obtain the projection of each evaluation value $r_{ij}^{(k)}$ on mean value r_{ij}^{*} by (13), for example,

$$\Pr j_{r_{11}^*} r_{11}^{(1)} = 0.647, \Pr j_{r_{11}^*} r_{11}^{(2)} = 0.698, \Pr j_{r_{11}^*} r_{11}^{(3)} = 0.596, \Pr j_{r_{11}^*} r_{11}^{(4)} = 0.639,$$

Table 2. The mean decision matrix $R*$

	G1	G2	G3
A1	$<[0.725,0.7875],[0.15,0.195]>$	$<[0.65,0.7],[0.225,0.275]>$	$<[0.745,0.8],[0.1425,0.175]>$
A2	$<[0.685,0.7375],[0.1775,0.225]>$	$<[0.7,0.75],[0.205,0.2375]>$	$<[0.75,0.7875],[0.125,0.1625]>$
A3	$<[0.69,0.7375],[0.185,0.2175]>$	$<[0.775,0.8125],[0.125,0.175]>$	$<[0.6475,0.7],[0.2275,0.275]>$
A4	$<[0.7375,0.78],[0.1625,0.2075]>$	$<[0.675,0.735],[0.1925,0.24]>$	$<[0.655,0.71],[0.2025,0.2375]>$

then, the weight of $r_{11}^{(k)}$ can be defined by (14),

$$\lambda_{11}^{(1)} = 0.246, \lambda_{11}^{(2)} = 0.243, \lambda_{11}^{(3)} = 0.267, \lambda_{11}^{(4)} = 0.243,$$

these are the weights of the four experts for A_1 with respect to G_1. After that we derive

$$r_{11} = \omega_{11}^{(1)} r_{11}^{(1)} + \omega_{11}^{(2)} r_{11}^{(2)} + \omega_{11}^{(3)} r_{11}^{(3)} + \omega_{11}^{(4)} r_{11}^{(4)} =< [0.73, 0.79], [0.15.0.19] >.$$

In the same way, we can get all the other values to form the following collective decision matrix $\tilde{R} = (\tilde{r}_{ij})_{4\times3}$ (see Table 3).

Table 3. The collective decision matrix $\tilde{R}$

	G1	G2	G3
A1	$<[0.726,0.788[0.149,0.190]>$	$<[0.65,0.7],[0.22,0.27]>$	$<[0.75,0.8],[0.14,0.173]>$
A2	$<[0.69,0.74],[0.17,0.22]>$	$<[0.71,0.75],[0.198,0.23]>$	$<[0.75,0.788],[0.12,0.16]>$
A3	$<[0.698,0.76],[0.179,0.21]>$	$<[0.778,0.81],[0.12,0.17]>$	$<[0.65,0.7],[0.225,0.27]>$
A4	$<[0.744,0.786],[0.158,0.20]>$	$<[0.68,0.74],[0.187,0.235]>$	$<[0.65,0.71],[0.20,0.237]>$

For $p = q = \frac{1}{2}$ from (9) and (10) or (11), we determine the weight of each criteri-
on $\omega_j (j = 1, 2, ..., n)$ as follow:

$$\omega_1 = 0.354, \omega_2 = 0.292, \omega_3 = 0.354.$$

Next, we calculate each line of to the decision matrix $\tilde{R}$ by the weighted arithmetic averaging operator (8) to get the comprehensive evaluation value of each alternative:

$$z_1(\omega) =< [0.715, 0.77], [0.163, 0.204] >, z_2(\omega) =< [0.46, 0.76], [0.157, 0.199] >,$$
$$z_3(\omega) =< [0.71, 0.76], [0.17, 0.216] >, z_4(\omega) =< [0.75, 0.75], [0.18, 0.22] >.$$

By the weighted score function formula (17), we obtain the weighted score function value of each alternative comprehensive evaluation value:

$$S(z_1)=0.559, S(z_2)=0.436, S(z_3)=0.54, S(z_3)=0.55,$$

That is $S(z_1)\succ S(z_4)\succ S(z_3)\succ S(z_2)$, where "$\succ$" indicates the relation "superior" or "preferred to", hence, the most desirable alternative is A_1.

Next, using the algorithm mentioned above we consider the case of all the expert weights are the same, then compare the results. That is to say, the weights of the four experts for A_i with respect to G_j are the same,

$$\lambda_{ij}^{(k)}=0.25(i=1,...4; j=1,...3; k=1,...4).$$

Then, we can get the collective decision matrix $\tilde{R}=(\tilde{r}_{ij})_{4\times 3}$ (see Table 4).

Table 4. The collective decision matrix $\tilde{R}$

	G1	G2	G3
A1	$<[0.725,0.7875],[0.15,0.195]>$	$<[0.65,0.7],[0.225,0.275]>$	$<[0.745,0.8],[0.1425,0.175]>$
A2	$<[0.685,0.7375],[0.1775,0.225]>$	$<[0.7,0.75],[0.205,0.2375]>$	$<[0.75,0.7875],[0.125,0.1625]>$
A3	$<[0.69,0.7375],[0.185,0.2175]>$	$<[0.775,0.8125],[0.125,0.175]>$	$<[0.6475,0.7],[0.2275,0.275]>$
A4	$<[0.7375,0.78],[0.1625,0.2075]>$	$<[0.675,0.735],[0.1925,0.24]>$	$<[0.655,0.71],[0.2025,0.2375]>$

Next, as the same way above we determine the weight of each criterion $\omega_j(j=1,2,\cdots,n)$ as following:

$$\omega_1=0.33, \omega_2=0.335, \omega_3=0.335.$$

Next, we calculate each line of to the decision matrix $\tilde{R}$ by the weighted arithmetic averaging operator (8) to get the comprehensive evaluation value of each alternative:

$$z_1(\omega)=<[0.71,0.77],[0.17,0.21]>, z_2(\omega)=<[0.71,0.76],[0.166,0.205]>,$$
$$z_3(\omega)=<[0.76,0.76],[0.17,0.22]>, z_4(\omega)=<[0.70,0.74],[0.185,0.228]>.$$

By the weighted score function formula (17), we obtain the weighted score function value of each alternative comprehensive evaluation value:

$$S(z_1)=0.55, S(z_2)=0.5495, S(z_3)=0.565, S(z_3)=0.5135,$$

That is $S(z_3)\succ S(z_1)\succ S(z_2)\succ S(z_4)$, so the most desirable alternative is A_3. It shows that we get a different result without regard to expert weight.

5 Conclusion

The previous study about multiple attribute group decision making problems mostly cope with the situation that the weights of the experts are determined beforehand or the weights of the experts are the same. In this paper, we have developed an algorithm to derive the weights of the experts from the decision matrices, and then different experts have different weights. This approach can avoid the affect of unfair evaluations, thus the result is more reasonable.

Competing Interests

Author has declared that no competing interests exist.

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