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Energy-Based $\mathcal{L}_2$-Disturbance Attenuation Control for Robot Manipulator With Uncertainties and Input Delay

Yong Ren[1], **Weiwei Sun**[1,2*] **and Baozeng Fu**[1]

[1] *Institute of Automation, Qufu Normal University, Qufu 273165, P.R.China*
[2] *School of Electrical Information and Automation, Qufu Normal University, Rizhao 276826, P.R.China*

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Abstract

This paper investigates the problem of $\mathcal{L}_2$-disturbance attenuation for robot manipulator with model uncertainty and input time delay. With the idea of shaping potential energy and the method of pre-feedback, a delayed Hamiltonian system structure is obtained for both full actuated and underactuated uncertain robot manipulator with time delay. Then the energy-based adaptive $\mathcal{L}_2$-disturbance attenuation controller is obtained by applying Lyapunov functional method. There spell out some sufficient conditions to guarantee the rationality and validity of the proposed control law. Simulation of a two-link robot manipulator is presented to illustrate the effectiveness of the achieved results in this paper.

Keywords: Hamiltonian system; robot manipulator; $\mathcal{L}_2$-disturbance attenuation; pre-feedback; time delay

1 Introduction

In recent years, robot technology has been drawn great attention in the field of high technology and a large number of scientific achievements are gained every year (see, *e.g.*, [1, 2, 3, 4, 5, 6] and the references therein). The operated robot is active mechanism. Each of its degrees of freedom (DOF) has a single drive. Manipulator system which stands for automatic control system with redundant, multivariate and the nonlinear in essential, is also a complex dynamic coupling system. Owing to each control problem of the system is equivalent to a dynamic problem, it is necessary to study the dynamic problem of the robot manipulator (see, *e.g.*, [7, 8, 9, 10] and the references therein). However, time delay can't be avoided in robot manipulator control area. Many factors, such as acquisition and transmission of the sensor signal, the calculation of the controller, actuation process of the actuator, can lead to time delay appears in the system. Time delay may degrade the performance of system

**Corresponding author: E-mail: wwsun@hotmail.com*

and lead to instability of the control system. Therefore, it is necessary to evaluate the impact of time delay on robot manipulator active control. The problem of robust force/position control for a robot manipulator had been investigated by using time-delay control method in [11]. In [12], decentralized linear time-invariant time-delayed joint controllers were designed for robot manipulator control. Using time delay control with gradient estimator, robust tracking of robot manipulator with nonlinear friction was studied in [13]. In addition, [14] considered the stability problem of a flexible-joint robot in case time delays were involved in the feedback loop.

In the past decades, energy-based control method has attracted considerable attention in the analysis and synthesis of nonlinear systems (see, *e.g.*, [15, 16, 17, 18] and the references therein). A key step in using energy-based control strategy is to express the system under consideration as a dissipative Hamiltonian system. This kind of system was put forward by [19]. The Hamilton function of Hamiltonian system can be taken as the sum of potential energy (excluding gravitational potential energy) and kinetic energy in physical systems, and it also can be regarded as a well candidate of Lyapunov function. Recently, the authors obtain some results on time delay Hamiltonian system (see, *e.g.*, [20, 21] and the references therein). In [21], sufficient conditions are derived and $\mathcal{L}_2$ feedback adaptive control law is designed for the time delay Hamiltonian system to guarantee the asymptotic stability and the $\mathcal{L}_2$ performance.

Energy-based control method is used in [22] to study the robot adaptive control of uncertain mechanical systems. Using the tool unified partial derivative operator (UPDO), an augment-ed Hamiltonian structure is provided and an adaptive $\mathcal{L}_2$-disturbance attenuation controller is designed for the mechanical systems. However, when delays inevitably appear, the augmented Hamiltonian structure and the control results in [22] may lose control. In view of the above, this paper extends the results of [22], which considers the $\mathcal{L}_2$-disturbance attenuation problem of n-DOF uncertainty robot manipulator with time delay in feedback. In order to use the energy-based control strategy, we firstly transform robot manipulator under consideration into delayed Hamiltonian system for both fully actuated and underactuated cases. It is easy to see that the matching condition in the underactated case is turned into a set of algebraic equations, which are much easier to cope with than a set of partial differential equations (PDEs) [23]. Then, we use the delayed Hamiltonian system to study energy-based robust adaptive control design problem for the uncertain robot manipulator system with time delays and a new adaptive $\mathcal{L}_2$-disturbance attenuation controller is designed. Finally, a simulation is given out to validate that there are robust properties, built-in capability in handling disturbances and uncertainties of the systems under the designed controller in this paper.

The rest of the paper is organized as follows. Section 2 presents the problem formulation and some preliminaries. In section 3, we study the augmented delayed Hamiltonian formulation for both fully actuated and underactuated uncertain robot manipulator system with time delays. The analysis of $\mathcal{L}_2$-disturbance attenuation of the delayed Hamiltonian system is presented in section 4. Section 5 illustrates the obtained results by a two-link robot manipulator example, which is followed by the conclusions in section 6.

Notations: $\mathbb{R}$ is the set of real numbers; $\mathbb{R}^n$ denotes the n-dimensional Euclidean space and $\mathbb{R}^{n\times m}$ is the real matrices with dimension $m \times n$; $\|\cdot\|$ stands for either the Euclidean vector norm or the induced matrix 2-norm. The notation $X \geq 0$ (respectively, $X > 0$) means that the matrix X is positive semidefinite (respectively, X is positive definite). $\otimes$ represents the Kronecker product. $A^{\perp}(\cdot)$ denotes the full rank left annihilator of matrix $A(\cdot)$, $A^{\perp}(\cdot)A(\cdot) = 0$. $F^{\dagger}$ denotes the pseudo-inverse of F, *i.e.*, $FF^{\dagger}F = F$.

2 Problem Statement and Preliminaries

Consider the following n-DOF robot manipulator described by the Euler-Lagrange equation [24]

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = B(q)\tau \tag{2.1}$$

where $q = \left[\begin{array}{c} q_1, q_2, \ldots, q_n \end{array} \right]^\mathsf{T} \in \mathbb{R}^n$ is the positive vector (the generalized coordinate), $\dot{q} \in \mathbb{R}^n$ is the velocity vector, $\tau \in \mathbb{R}^m$ is the control torque vector, $B(q) \in \mathbb{R}^{n\times m}$ has full column rank $(m \leq n)$, $M(q) \in \mathbb{R}^{n\times n}$ is the inertia matrix, $M(q) > 0$, $G(q) \in \mathbb{R}^n$ describes the potential forces and the matrix $C(q, \dot{q}) \in \mathbb{R}^{n\times n}$ describes the Coriolis and centripetal forces. $M(q)$, $C(q, \dot{q})$ and $G(q)$ are assumed to have unknown constant parameters.

We consider the existence of several time delays in the input signals applied to the robot joints. Let $h \geq 0$ is the time delay involved in every component of the input vector. The input delay is denoted by $\tau(t, t-h)$ which illustrates the impact of local input signals and remote signals. Thus, under the presence of time delays and disturbances, the control model of the robot manipulator (2.1) can be rewritten as follows

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = B(q)\tau(t, t-h) + \omega \tag{2.2}$$

where $\omega \in \mathbb{R}^n$ is the external disturbance.

Assumption 2.1. *The unknown part in $G(q)$ depends linearly on a constant vector $\theta \in \mathbb{R}^{n\times l}$,* i.e.*, there exists a matrix $\Phi(q) \in \mathbb{R}^{n\times l}$ such that*

$$G(q) = G_0(q) + \Phi(q)\theta \tag{2.3}$$

where $G_0(q) \in \mathbb{R}^n$ is the separable known or nominal part of $G(q)$.

Our aim in this paper is to investigate the $\mathcal{L}_2$-disturbance attenuation problem of the system (2.2) under Hamiltonian systems framework. Thus, firstly, we should transform (2.2) into a time delayed Hamiltonian system.

Under Assumption 2.1, we consider a Hamilton function for the system (2.2)

$$H(q, p, \hat{\theta}) = K(q, p) + P_g(q) + \frac{1}{2}(\hat{\theta} - \theta)^\mathsf{T}\Gamma_0(\hat{\theta} - \theta) \tag{2.4}$$

where

$$K(q, p) := \frac{1}{2}p^\mathsf{T}M^{-1}(q)p = \frac{1}{2}\dot{q}^\mathsf{T}M(q)\dot{q} \tag{2.5}$$

is the system's kinetic energy,

$$P_g(q) := \frac{1}{2}(q - q^0)^\mathsf{T}\Lambda(q - q^0) \tag{2.6}$$

is the so-called virtual potential energy, $\Lambda \in \mathbb{R}^{n\times n}$ and $\Gamma_0 \in \mathbb{R}^{l\times l}$ are two constant positive definite matrices; $\hat{\theta}$ is the estimate of θ, $q^0 \in \mathbb{R}^n$ is the target position which is to be designed, $p \in \mathbb{R}^n$ is the generalized momenta.

Obviously, we have

$$\frac{\partial H(q, p, \hat{\theta})}{\partial p} = M^{-1}(q)p = \dot{q} \tag{2.7}$$

which means

$$p = M(q)\dot{q}. \tag{2.8}$$

The following lemma is necessary for Hamiltonian modelling of system (2.2).

Lemma 2.1. *(*[22]) Assume that $A(x) \in \mathbb{R}^{n\times n}(x \in \mathbb{R}^n)$ is a function matrix, $\alpha, \beta \in \mathbb{R}^n$ are constant vectors. Then,

$$\frac{\partial(\alpha^\mathsf{T}A(x)\beta)}{\partial x} = (I_n \otimes \alpha^\mathsf{T})(\Gamma_n \cdot \frac{\partial A(x)}{\partial x})\beta$$

where $\frac{\partial A(x)}{\partial x} = A(x) \otimes \frac{\partial}{\partial x}$ and $\Gamma_n = \prod_{i=1}^{n-1}\prod_{j>i}^{n} E_{n^2}((i-1)n+j), (j-1)n+i)$, $E_{n^2}((i-1)n+j), (j-1)n+i) \in \mathbb{R}^{n^2\times n^2}$ is obtained by swapping the $((i-1)n+j)$th row with the $((j-1)n+i)$th row of the $n^2 \times n^2$ identity matrix I_{n^2}.

Using Lemma 2.1, we can get

$$\frac{\partial H(q,p,\hat{\theta})}{\partial q} = \frac{1}{2}(I_n \otimes p^{\mathsf{T}})(\Gamma_n \frac{\partial M^{-1}}{\partial q})p + \Lambda(q - q^0). \tag{2.9}$$

From system (2.2) with $\omega = 0$ and (2.8), the derivative of p along the time t satisfies

$$\begin{aligned} \dot{p} &= \dot{M}(q)\dot{q} + M(q)\ddot{q} \\ &= \dot{M}(q)\dot{q} - C(q,\dot{q})\dot{q} - G(q) + B(q)\tau(t, t-h). \end{aligned} \tag{2.10}$$

3 Delayed Hamiltonian Formulation

In this section, we consider the Hamiltonian formulation problem for system (2.2) in two cases, *i.e.*, 1) the system is fully actuated $(m = n)$; 2) the system is underactuated $(m < n)$. Without loss of generality, we assume $\omega = 0$ in system (2.2).

3.1 Fully Actuated Case

In order to get a nice Hamiltonian structure for system (2.2), we design a pre-feedback law as follows

$$\begin{cases} \tau(t, t-h) = B^{-1}(q)[G_0(q) + \frac{1}{2}\Phi(q)\hat{\theta} - \Lambda(q - q^0) - K_D\dot{q}(t-h) \\ \qquad\qquad + \frac{1}{2}\Phi(q)\hat{\theta}(t-h) - K_{D1}\dot{q}] + u, \\ \dot{\hat{\theta}} = -\frac{1}{2}\Gamma_0^{-1}\Phi^{\mathsf{T}}(q)\dot{q} - \frac{1}{2}\Gamma_0^{-1}\Phi^{\mathsf{T}}(q)\dot{q}(t-h), \\ \hat{\theta}(t) = \varphi(t), \quad t \in [-h, 0] \end{cases} \tag{3.1}$$

where $K_D,\ K_{D1} \in \mathbb{R}^{n\times n}$ are both constant positive definite matrices to be determined, u is the new control input, $\varphi(t)$ is a continuous vector-valued initial function.

From (2.7)-(3.1), we can get

$$\begin{aligned} \begin{bmatrix} \dot{q} \\ \dot{p} \\ \dot{\hat{\theta}} \end{bmatrix} &= \begin{bmatrix} 0 & I_n & 0 \\ -I_n & K_c(q,p) - K_{D1} & \frac{1}{2}\Phi(q){\Gamma_0}^{-1} \\ 0 & -\frac{1}{2}{\Gamma_0}^{-1}\Phi^{\mathsf{T}}(q) & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial H(q,p,\hat{\theta})}{\partial q} \\ \frac{\partial H(q,p,\hat{\theta})}{\partial p} \\ \frac{\partial H(q,p,\hat{\theta})}{\partial \hat{\theta}} \end{bmatrix} \\ &\quad + \begin{bmatrix} 0 & 0 & 0 \\ 0 & -K_D & \frac{1}{2}\Phi(q){\Gamma_0}^{-1} \\ 0 & -\frac{1}{2}{\Gamma_0}^{-1}\Phi^{\mathsf{T}}(q) & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial H(q(t-h),p(t-h),\hat{\theta}(t-h))}{\partial q} \\ \frac{\partial H(q(t-h),p(t-h),\hat{\theta}(t-h))}{\partial p} \\ \frac{\partial H(q(t-h),p(t-h),\hat{\theta}(t-h))}{\partial \hat{\theta}} \end{bmatrix} \\ &\quad + \begin{bmatrix} 0 \\ B(q) \\ 0 \end{bmatrix} u \end{aligned} \tag{3.2}$$

where

$$K_c(q,p) = \dot{M}(q) - C(q,\dot{q}) + \frac{1}{2}(I_n \otimes P^{\mathsf{T}})(\Gamma_n \frac{\partial M^{-1}(q)}{\partial q})M(q). \tag{3.3}$$

Since

$$\frac{\partial}{\partial x}[A^{-1}(x)] = -\Gamma_n(I_n \otimes A^{-1}(x))(\Gamma_n \frac{\partial A(x)}{\partial x})A^{-1}(x)$$

holds, we can prove that $K_c(q,p) \equiv 0$ by using the properties of the Kronecker product. The details of the proof can be found in [22]. Here, they are omitted for the sake of brevity.

Let

$$X = \begin{bmatrix} q \\ p \\ \hat{\theta} \end{bmatrix} \in \mathbb{R}^{2n+l},$$

then, system(2.2) ($\omega = 0$) can be transformed into the following delayed port-controlled Hamiltonian system

$$\begin{cases} \dot{X} = [J(X) - R(X)]\dfrac{\partial H(X(t))}{\partial X} + [J_1(X) - R_1(X)]\dfrac{\partial H(X(t-h))}{\partial X} + g_c u, \\ X(t) = \phi(t), \;\; t \in [-h, 0] \end{cases} \tag{3.4}$$

where $\phi(t)$ is a continuous vector-valued initial function,

$$\begin{aligned}
J(X) &= \begin{bmatrix} 0 & I_n & 0 \\ -I_n & 0 & \frac{1}{2}\Phi(q){\Gamma_0}^{-1} \\ 0 & -\frac{1}{2}{\Gamma_0}^{-1}\Phi^{\mathsf{T}}(q) & 0 \end{bmatrix}, \\
J_1(X) &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2}\Phi(q){\Gamma_0}^{-1} \\ 0 & -\frac{1}{2}{\Gamma_0}^{-1}\Phi^{\mathsf{T}}(q) & 0 \end{bmatrix}, \\
R(X) &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & K_{D1} & 0 \\ 0 & 0 & 0 \end{bmatrix} \in \mathbb{R}^{(2n+l)\times(2n+l)}, \\
R_1(X) &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & K_D & 0 \\ 0 & 0 & 0 \end{bmatrix} \in \mathbb{R}^{(2n+l)\times(2n+l)}, \\
g_c &= \begin{bmatrix} 0 \\ B(q) \\ 0 \end{bmatrix} \in \mathbb{R}^{(2n+l)\times m}.
\end{aligned}$$

Obviously, $J(X) = -J^{\mathsf{T}}(X)$, $J_1(X) = -J_1^{\mathsf{T}}(X)$, $R(X) \geq 0$, $R_1(X) \geq 0$.

We may summarize the above analysis as follows.

Theorem 3.1. *Consider fully-actuated robot system (2.2) with $\omega = 0$ and Assumption 2.1 holds. With the Hamilton function (2.4) and the adaptive pre-feedback law (3.1), system (2.2) can be transformed into a delayed Hamiltonian system described as (3.4).*

3.2 Underactuated Case

In this case, $B(q)$ is singular. In order to get a nice Hamiltonian structure, a suitable pre-feedback law $\tau(q,p,\hat{\theta},\dot{q},\dot{q}(t-h),\hat{\theta}(t-h))$ should be designed for system (2.2) with $\omega = 0$ such that

$$\begin{aligned}
& B(q)\tau(q,p,\hat{\theta},\dot{q},\dot{q}(t-h),\hat{\theta}(t-h)) \\
= \;& G_0(q) + \frac{1}{2}\Phi(q)\hat{\theta} - \Lambda(q-q^0) - K_D\dot{q}(t-h) + \frac{1}{2}\Phi(q)\hat{\theta}(t-h) - K_{D1}\dot{q} \\
& + B(q)u(q,p,\hat{\theta},\dot{q},\dot{q}(t-h),\hat{\theta}(t-h)),
\end{aligned} \tag{3.5}$$

where $K_D, \; K_{D1} \in \mathbb{R}^{n\times n}$ are both constant positive define matrixes to be determined.

Due to $B(q)$ is singular and has full column rank, τ can only affect the terms in the range space of $B(q)$. Thus, if the following equation

$$B^{\perp}(q)[G_0(q) + \frac{1}{2}\Phi(q)\hat{\theta} - \Lambda(q - q^0) - K_D\dot{q}(t-h) + \frac{1}{2}\Phi(q)\hat{\theta}(t-h) - K_{D1}\dot{q}] = 0 \tag{3.6}$$

holds, then the equation (3.5) will always holds for any choice of τ.

We may solve the equation (3.6) to obtain the solution group (Λ, K_D, K_{D1}) and then express the polynomial

$$G_0(q) + \frac{1}{2}\Phi(q)\hat{\theta} - \Lambda(q - q^0) - K_D\dot{q}(t-h) + \frac{1}{2}\Phi(q)\hat{\theta}(t-h) - K_{D1}\dot{q}$$

as follows

$$\begin{aligned} & G_0(q) + \frac{1}{2}\Phi(q)\hat{\theta} - \Lambda(q - q^0) - K_D\dot{q}(t-h) + \frac{1}{2}\Phi(q)\hat{\theta}(t-h) - K_{D1}\dot{q} \\ = & \sum_{i=1}^{m} a_i(q, p, \hat{\theta}, \dot{q}, \dot{q}(t-h), \hat{\theta}(t-h))\alpha_i(q) \end{aligned} \tag{3.7}$$

where $a_i(q, p, \hat{\theta}, \ldots, \dot{q}(t-h), \hat{\theta}(t-h)), i = 1, 2, \ldots, m$ are scalar functions; $\alpha_1(q),\ldots,\alpha_m(q)$ are column vectors of $B(q)$, *i.e.*, $B(q) = [\alpha_1(q), \ldots, \alpha_m(q)]$.

Based on the above analysis, we can choose an adaptive pre-feedback law as follows

$$\begin{cases} \tau(t, t-h) = \tau(q, p, \hat{\theta}, \dot{q}, \dot{q}(t-h), \hat{\theta}(t-h)) \\ \qquad = [a_1(q, p, \hat{\theta}, \dot{q}, \dot{q}(t-h), \hat{\theta}(t-h)), \ldots, a_m(q, p, \hat{\theta}, \dot{q}, \dot{q}(t-h), \hat{\theta}(t-h))]^{\mathsf{T}} + u, \\ \dot{\hat{\theta}} = -\frac{1}{2}\Gamma_0^{-1}\Phi^{\mathsf{T}}(q)\dot{q} - \frac{1}{2}\Gamma_0^{-1}\Phi^{\mathsf{T}}(q)\dot{q}(t-h), \\ \hat{\theta}(t) = \varphi(t), \quad t \in [-h, 0] \end{cases} \tag{3.8}$$

where u is the new control input.

Thus, we have

$$B(q)\tau = G_0(q) + \frac{1}{2}\Phi(q)\hat{\theta} - \Lambda(q - q^0) - K_D\dot{q}(t-h) + \frac{1}{2}\Phi(q)\hat{\theta}(t-h) - K_{D1}\dot{q} + B(q)u. \tag{3.9}$$

From (2.4)-(2.10) and (3.8), underactuated system (2.2) with $\omega = 0$ can be expressed the same as (3.2).

Furthermore, we can also get $K_c \equiv 0$ and thus system (2.2) ($\omega = 0$) can be transformed into the delayed Hamiltonian system described as (3.4).

Theorem 3.2. *Consider underactuated robot system (2.2) with $\omega = 0$ and Assumption 2.1 holds. With the Hamilton function (2.4) and the adaptive pre-feedback law (3.8), system (2.2) can be transformed into a delayed Hamiltonian system described as (3.4).*

From the above analysis, we can get that whether the robot system (2.2) is fully actuated or underactuated, it has the same Hamiltonian formulation (3.4).

4 Energy-Based $\mathcal{L}_2$-Disturbance Attenuation

In this section, we will study energy-based robust adaptive control of uncertain robot manipulator system (2.2) by using the obtained delayed Hamiltonian system formulation, and design an adaptive $\mathcal{L}_2$-disturbance attenuation controller for system (2.2).

Let

$$z = h(X)g^{\mathsf{T}}(X)\frac{\partial H(X)}{\partial X} \tag{4.1}$$

where $z \in \mathbb{R}^q$ is the penalty signal, $h(X) \in \mathbb{R}^{q\times m}$ is the weighting matrix with full column rank.

According to Theorem 3.1 and Theorem 3.2 in section 3, under the following control law

$$\begin{cases} \tau(t,t-h) = W(q,p,\hat{\theta},\dot{q},\dot{q}(t-h),\hat{\theta}(t-h)) + u, \ \ when\ m<n \\ \tau(t,t-h) = B^{-1}(q)[G_0(q) + \frac{1}{2}\Phi(q)\hat{\theta} - \Lambda(q-q^0) - K_D\dot{q}(t-h) \\ \qquad\qquad + \frac{1}{2}\Phi(q)\hat{\theta}(t-h) - K_{D1}\dot{q}] + u, \ \ when\ m=n \\ \dot{\hat{\theta}} = -\frac{1}{2}\Gamma_0^{-1}\Phi^{\mathsf{T}}(q)\dot{q} - \frac{1}{2}\Gamma_0^{-1}\Phi^{\mathsf{T}}(q)\dot{q}(t-h), \\ \hat{\theta}(t) = \varphi(t), \ \ t\in[-h,0], \end{cases} \tag{4.2}$$

system (2.2) can be transformed into

$$\begin{cases} \dot{X} = (J(X)-R(X))\dfrac{\partial H(X)}{\partial X} + (J_1(X)-R_1(X))\dfrac{\partial H(X(t-h))}{\partial X} + g_c u + g_d\omega, \\ z = h(X)g_c^{\mathsf{T}}\dfrac{\partial H(X)}{\partial X} \end{cases} \tag{4.3}$$

where $g_d = [0\ I_n\ 0]^{\mathsf{T}}$, X, $J(X)$, $R(X)$, $J_1(X)$, $R_1(X)$, $H(X)$, g_c, K_D, K_{D1} have the same definitions as above,

$$\begin{aligned} & W(q,p,\hat{\theta},\dot{q},\dot{q}(t-h),\hat{\theta}(t-h)) \\ = \ & [a_1(q,p,\hat{\theta},\dot{q},\dot{q}(t-h),\hat{\theta}(t-h)),\ldots,a_m(q,p,\hat{\theta},\dot{q},\dot{q}(t-h),\hat{\theta}(t-h))]. \end{aligned} \tag{4.4}$$

The following Lemma is necessary for the analysis of $\mathcal{L}_2$-disturbance attenuation problem of (4.3).

Lemma 4.1. [25] *For any vectors a, $b \in \mathbb{R}^n$ and a matrix $M > 0$ with compatible dimensions, the following inequality holds*

$$2a^{\mathsf{T}}b \le a^{\mathsf{T}}Ma + b^{\mathsf{T}}M^{-1}b. \tag{4.5}$$

For a given disturbance attention level $\gamma > 0$ and the penalty signal described as (4.1), we have the following result.

Theorem 4.1. *For a given disturbance attenuation level $\gamma > 0$, if*

$$R(X) - \frac{1}{2\gamma^2}g_d g_d^{\mathsf{T}} \ge 0 \tag{4.6}$$

holds, then the $\mathcal{L}_2$-disturbance attenuation problem of system (4.3) can be solved under the feedback control law

$$u = -\frac{1}{2}\Big\{g_c^{\dagger}[Q + (J_1(X)-R_1(X))Q^{-1}(J_1(X)-R_1(X))^{\mathsf{T}}] + h(X)^{\mathsf{T}}h(X)g_c^{\mathsf{T}}\Big\}\frac{\partial H(X)}{\partial X} \tag{4.7}$$

and the γ-dissipation inequality is

$$\dot{V}(X(t-h),t) + \frac{\partial^{\mathsf{T}}H(X)}{\partial X}[R(X) - \frac{1}{2\gamma^2}g_d g_d^{\mathsf{T}}]\frac{\partial H(X)}{\partial X} \le \frac{1}{2}(\gamma^2\|\omega\|^2 - \|z\|^2)$$

where

$$V(X(t-h),t) = H(X(t)) + \frac{1}{2}\int_{t-h}^{t}\frac{\partial^{\mathsf{T}}H(X(s))}{\partial X}Q\frac{\partial H(X(s))}{\partial X}ds, \tag{4.8}$$

Q is a positive definite matrix.

Proof. Substituting (4.7) into (4.3), we get the closed-loop system of (4.3),

$$\begin{cases} \dot{X} = (J(X)-R(X))\dfrac{\partial H(X)}{\partial X} + (J_1(X)-R_1(X))\dfrac{\partial H(X(t-h))}{\partial X} \\ \qquad - \frac{1}{2}[Q + (J_1(X)-R_1(X))Q^{-1}(J_1(X)-R_1(X))^{\mathsf{T}} + g_c h^{\mathsf{T}}(X)h(X)g_c^{\mathsf{T}}]\dfrac{\partial H(X)}{\partial X} + g_d\omega, \\ z = h(X)g_c^{\mathsf{T}}\dfrac{\partial H(X)}{\partial X} \end{cases} \tag{4.9}$$

Choose a Lyapunov functional described as (4.8) and computing the derivative of $V(X(t-h),t)$ along the trajectory of the closed-loop system (4.9), and based on Lemma 4.1, we have

$$\begin{aligned}
& \dot{V}(X(t-h),t) \\
= \;& \frac{\partial^{\mathsf T} H(X)}{\partial X}\dot{X} + \frac{1}{2}\frac{\partial^{\mathsf T} H(X)}{\partial X}Q\frac{\partial H(X)}{\partial X} - \frac{1}{2}\frac{\partial^{\mathsf T} H(X(t-h))}{\partial X}Q\frac{\partial H(X(t-h))}{\partial X} \\
= \;& \frac{\partial^{\mathsf T} H(X)}{\partial X}(J(X)-R(X))\frac{\partial H(X)}{\partial X} + \frac{\partial^{\mathsf T} H(X)}{\partial X}(J_1(X)-R_1(X))\frac{\partial H(X(t-h))}{\partial X} \\
& + \frac{\partial^{\mathsf T} H(X)}{\partial X}g_c u + \frac{\partial^{\mathsf T} H(X)}{\partial X}g_d\omega - \frac{1}{2}\frac{\partial^{\mathsf T} H(X(t-h))}{\partial X}Q\frac{\partial H(X(t-h))}{\partial X} \\
& + \frac{1}{2}\frac{\partial^{\mathsf T} H(X)}{\partial X}Q\frac{\partial H(X)}{\partial X} \\
\leq \;& -\frac{\partial^{\mathsf T} H(X)}{\partial X}R(X)\frac{\partial H(X)}{\partial X} + \frac{1}{2}\frac{\partial^{\mathsf T} H(X)}{\partial X}(J_1(X)-R_1(X))Q^{-1}(J_1(X)-R_1(X))^{\mathsf T} \\
& \times\frac{\partial H(X)}{\partial X} + \frac{1}{2}\frac{\partial^{\mathsf T} H(X(t-h))}{\partial X}Q\frac{\partial H(X(t-h))}{\partial X} + \frac{\partial^{\mathsf T} H(X)}{\partial X}g_c\Big\{-\frac{1}{2}[g_c^{\dagger}(Q \\
& + (J_1(X)-R_1(X))Q^{-1}(J_1(X)-R_1(X))^{\mathsf T}) + h^{\mathsf T}(X)h(X)g_c^{\mathsf T}]\frac{\partial H(X)}{\partial X}\Big\} \\
& + \frac{\partial^{\mathsf T} H(X)}{\partial X}g_d\omega + \frac{1}{2}\frac{\partial^{\mathsf T} H(X)}{\partial X}Q\frac{\partial H(X)}{\partial X} - \frac{1}{2}\frac{\partial^{\mathsf T} H(X(t-h))}{\partial X}Q\frac{\partial H(X(t-h))}{\partial X} \\
\leq \;& -\frac{\partial^{\mathsf T} H(X)}{\partial X}R(X)\frac{\partial H(X)}{\partial X} + \frac{1}{2\gamma^2}\frac{\partial^{\mathsf T} H(X)}{\partial X}g_d g_d^{\mathsf T}\frac{\partial H(X)}{\partial X} \\
& + \frac{1}{2}(\gamma^2 \parallel \omega \parallel^2 - \parallel z \parallel^2) - \frac{1}{2}\parallel \gamma\omega - \frac{1}{\gamma}g_d^{\mathsf T}\frac{\partial H(X)}{\partial X}\parallel^2
\end{aligned} \tag{4.10}$$

therefore,

$$\begin{aligned}
& \dot{V}(X(t-h),t) + \frac{\partial^{\mathsf T} H(X)}{\partial X}(R(X) - \frac{1}{2\gamma^2}g_d g_d^{\mathsf T})\frac{\partial H(X)}{\partial X} \\
& \leq \frac{1}{2}(\gamma^2 \parallel \omega \parallel^2 - \parallel z \parallel^2) - \frac{1}{2}\parallel \gamma\omega - \frac{1}{\gamma}g_d^{\mathsf T}\frac{\partial H(X)}{\partial X}\parallel^2 \\
& \leq \frac{1}{2}(\gamma^2 \parallel \omega \parallel^2 - \parallel z \parallel^2).
\end{aligned} \tag{4.11}$$

Since (4.6) holds, $V(X(t-h),t)$ is a solution to the $\mathcal{L}_2$-disturbance attenuation of system (4.3). □

Based on Theorem 4.1, substituting (4.7) into (3.8), we can get a complete adaptive $\mathcal{L}_2$-disturbance attenuation controller for system (2.2) as follows

$$\begin{cases}
\tau(t,t-h) = W(q,p,\hat{\theta},\dot{q},\dot{q}(t-h),\hat{\theta}(t-h)) \\
\qquad - \frac{1}{2}[g_c^{\dagger}(Q+[J_1-R_1]Q^{-1}[J_1-R_1]^{\mathsf T}) + h^{\mathsf T}hg_c^{\mathsf T}]\frac{\partial H(X)}{\partial X}, \;\; when \;\; m<n \\
\tau(t,t-h) = B^{-1}(q)[G_0(q) + \frac{1}{2}\Phi(q)\hat{\theta} - \Lambda(q-q^0) - K_D\dot{q}(t-h) + \frac{1}{2}\Phi(q)\hat{\theta}(t-h) - K_{D1}\dot{q}] \\
\qquad - \frac{1}{2}[g_c^{\dagger}(Q+[J_1-R_1]Q^{-1}[J_1-R_1]^{\mathsf T}) + h^{\mathsf T}hg_c^{\mathsf T}]\frac{\partial H(X)}{\partial X}, \;\; when \;\; m=n \\
\dot{\hat{\theta}} = -\frac{1}{2}\Gamma_0^{-1}\Phi^{\mathsf T}(q)\dot{q} - \frac{1}{2}\Gamma_0^{-1}\Phi^{\mathsf T}(q)\dot{q}(t-h), \\
\hat{\theta}(t) = \varphi(t), \;\; t\in[-h,0]
\end{cases} \tag{4.12}$$

where $J_1 := J_1(X), R_1 := R_1(X)$.

Thus, we can gain the following result for system (2.2).

Theorem 4.2. *Consider robot manipulator (2.2) and Assumption 2.1 holds. q^0 is the target point of system (2.2). The solution group (Λ, K_D, K_{D1}) of the constraint equation (3.6) can be gotten when* *the system is underactuated. For the given γ, if $R(X) - \frac{1}{2\gamma^2} g_d g_d^{\mathsf{T}} \geq 0$ holds, then an adaptive $\mathcal{L}_2$-disturbance attenuation controller can be designed as (4.12) for system (2.2).*

5 Illustrative Example

In this section, we give an example to show that 1) how to transform the robot manipulator with time delay into delayed Hamiltonian system; and 2) how to design an energy-based adaptive $\mathcal{L}_2$-disturbance attention controller for the delayed robot system under Hamiltonian system framework.

A planar two-link manipulator with two nodes in the vertical plane is considered as shown in Figure 1, where we assume that the mass m_p of payload is unknown, m_i and l_i are the mass and length of link i, l_{ci} is the distance from node $i-1$ to the center of mass of link i, I_i is the moment of inertia of link i about an axis coming to page through the center of the mass of link i, $i = 1, 2$ [7].

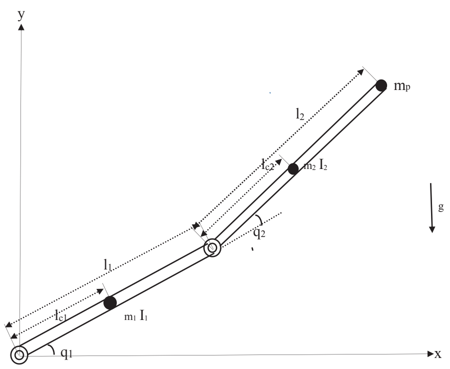

Figure 1: Planar two-link manipulator with payload.

We assume the existence of delay in the input signals,

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau(t, t-h) + \omega \tag{5.1}$$

where $q = [q_1, q_2]^{\mathsf{T}} \in \mathbb{R}^2$ is the angular position vector, $\tau(t, t-h)$ is the control torque, $\omega \in \mathbb{R}^2$ is the disturbance.

$$\begin{aligned}
M(q) &= \begin{bmatrix} \overline{m}_1 + \overline{m}_2 + 2\overline{m}_3 \cos q_2 & \overline{m}_2 + \overline{m}_3 \cos q_2 \\ \overline{m}_2 + \overline{m}_3 \cos q_2 & \overline{m}_2 \end{bmatrix}, \\
C(q,\dot{q}) &= \begin{bmatrix} -\overline{m}_3 \dot{q}_2 sin q_2 & -\overline{m}_3(\dot{q}_1 + \dot{q}_2) sin q_2 \\ \overline{m}_3 \dot{q}_1 sin q_2 & 0 \end{bmatrix}, \\
G(q) &= \begin{bmatrix} \overline{m}_4 g \cos q_1 + \overline{m}_5 g \cos(q_1 + q_2) \\ \overline{m}_5 g \cos(q_1 + q_2) \end{bmatrix}, \\
\overline{m}_1 &= m_1 l_{c1}^2 + m_2 l_1^2 + I_1 + m_p l_1^2, \\
\overline{m}_2 &= m_2 l_{c2}^2 + I_2 + m_p l_2^2, \\
\overline{m}_3 &= m_2 l_1 l_{c2} + m_p l_1 l_2, \\
\overline{m}_4 &= m_1 l_{c2} + m_2 l_1 + m_p l_1, \\
\overline{m}_5 &= m_2 l_{c2} + m_p l_2.
\end{aligned}$$

Due to the payload's mass m_p is unknown, we can see that $M(q)$, $C(q,\dot{q})$ and $G(q)$ are not exactly known. Next, we transform system (5.1) into a delayed Hamiltonian system according to Theorem 3.1.

Let $\theta := m_p$, which denotes the unknown parameter, then $G(q)$ can be written as

$$G(q) = G_0(q) + \Phi(q)\theta \tag{5.2}$$

where

$$G_0(q) = \begin{bmatrix} (m_1 l_{c2} + m_2 l_1) g \cos q_1 + m_2 l_{c2} g \cos(q_1 + q_2) \\ m_2 l_{c2} g \cos(q_1 + q_2) \end{bmatrix}, \tag{5.3}$$

$$\Phi(q) = \begin{bmatrix} l_1 g \cos(q_1) + l_2 g \cos(q_1 + q_2) \\ l_2 g \cos(q_1 + q_2) \end{bmatrix} := \begin{bmatrix} \phi_1(q) \\ \phi_2(q) \end{bmatrix}. \tag{5.4}$$

We consider $q^0 = [q_1^0, q_2^0]^\mathsf{T} \in \mathbb{R}^2$ as the target position of the system which is to be designed. $p = [p_1, p_2]^\mathsf{T} = M(q)\dot{q}, \Lambda = Diag\{\lambda_1, \lambda_2\} > 0$ and $\Gamma_0 = \lambda_3 > 0$. Consider

$$\begin{aligned} H(q,p,\hat{\theta}) &= K(q,p) + P_g(q) + \frac{1}{2}(\hat{\theta} - \theta)^\mathsf{T} \Gamma_0 (\hat{\theta} - \theta) \\ &= \frac{1}{2} p^\mathsf{T} M^{-1}(q) p + \frac{1}{2}(q - q^0)^\mathsf{T} \Lambda (q - q^0) + \frac{\Gamma_0}{2}(\hat{\theta} - \theta)^2 \end{aligned} \tag{5.5}$$

as the Hamilton function and

$$\dot{\hat{\theta}} = -\frac{1}{2}\Gamma_0^{-1}\Phi^\mathsf{T}(q)\dot{q} - \frac{1}{2}\Gamma_0^{-1}\Phi^\mathsf{T}(q)\dot{q}(t-h). \tag{5.6}$$

The pre-feedback law can be designed as follows

$$\begin{cases} \tau(t,t-h) = G_0(q) + \frac{1}{2}\Phi(q)\hat{\theta} - \Lambda(q - q^0) - K_D \dot{q}(t-h) \\ \qquad\qquad + \frac{1}{2}\Phi(q)\hat{\theta}(t-h) - K_{D1}\dot{q} + u, \\ \dot{\hat{\theta}} = -\frac{1}{2}\Gamma_0^{-1}\Phi^\mathsf{T}(q)\dot{q} - \frac{1}{2}\Gamma_0^{-1}\Phi^\mathsf{T}(q)\dot{q}(t-h), \\ \hat{\theta}(t) = \varphi(t), \quad t \in [-h, 0] \end{cases} \tag{5.7}$$

where $K_D = Diag\{k_{d1}, k_{d2}\} > 0$, $K_{D1} = Diag\{k_{d11}, k_{d22}\} > 0$.

According to Theorem 3.1, by the Hamilton function (5.5) and the pre-feedback law (5.7), system (5.1) can be transformed into the following delayed Hamiltonian system

$$\begin{cases} \dot{X} = [J(X) - R(X)]\dfrac{\partial H(X)}{\partial X} + [J_1(X) - R_1(X)]\dfrac{\partial H(X(t-h))}{\partial X} + g_c u + g_d \omega, \\ X(t) = \phi(t), \quad t \in [-h, 0] \end{cases} \tag{5.8}$$

where

$$
\begin{aligned}
X &= \left[\ q_1, q_2, p_1, p_2, \hat{\theta}\ \right]^{\mathsf{T}} \in \mathbb{R}^5, \\
J(X) &= \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & \frac{\phi_1(q)}{2\lambda_3} \\ 0 & -1 & 0 & 0 & \frac{\phi_2(q)}{2\lambda_3} \\ 0 & 0 & -\frac{\phi_1(q)}{2\lambda_3} & -\frac{\phi_2(q)}{2\lambda_3} & 0 \end{bmatrix}, \\
J_1(X) &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\phi_1(q)}{2\lambda_3} \\ 0 & 0 & 0 & 0 & \frac{\phi_2(q)}{2\lambda_3} \\ 0 & 0 & -\frac{\phi_1(q)}{2\lambda_3} & -\frac{\phi_2(q)}{2\lambda_3} & 0 \end{bmatrix}, \\
R(X) &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & k_{d1} & 0 & 0 \\ 0 & 0 & 0 & k_{d2} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \\
R_1(X) &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & k_{d11} & 0 & 0 \\ 0 & 0 & 0 & k_{d22} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \\
g_c &= g_d = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}.
\end{aligned}
$$

Obviously, $J(X) = -J^{\mathsf{T}}(X)$, $J_1(X) = -J_1^{\mathsf{T}}(X)$, $R(X) \geq 0, R_1(X) \geq 0$.

Furthermore, let $\gamma^2 > max\{k_{d1}, k_{d2}\}$, then $R(x) - \frac{1}{2\gamma^2} g_d g_d^{\mathsf{T}} \geq 0$ holds. Thus, an energy-based adaptive $\mathcal{L}_2$-disturbance attention controller of system (5.1) can be achieved according to Theorem 4.1 as follows

$$
\begin{cases}
\tau(t, t-h) = G_0(q) + \frac{1}{2}\Phi(q)\hat{\theta} - \Lambda(q - q^0) - K_D\dot{q}(t-h) + \frac{1}{2}\Phi(q)\hat{\theta}(t-h) - K_{D1}\dot{q} \\
\qquad\qquad - \frac{1}{2}[g_c^{\dagger}(Q + [J_1(X) - R_1(X)]Q^{-1}[J_1(X) - R_1(X)]^{\mathsf{T}}) + h(X)^{\mathsf{T}} h(X) g_c^{\mathsf{T}}] \dfrac{\partial H}{\partial X}, \\
\dot{\hat{\theta}} = -\frac{1}{2}\Gamma_0^{-1}\Phi^{\mathsf{T}}(q)\dot{q} - \frac{1}{2}\Gamma_0^{-1}\Phi^{\mathsf{T}}(q)\dot{q}(t-h), \\
\hat{\theta}(t) = \varphi(t), \quad t \in [-h, 0]
\end{cases}
\tag{5.9}
$$

i.e.,

$$
\begin{cases}
\tau_1(t, t-h) = (m_1 l_{c2}) + m_2 l_1) g \cos q_1 + m_2 l_{c2} g \cos(q_1 + q_2) - \lambda_1(q_1 - q_1^0) \\
\qquad\qquad - k_{d11}\dot{q}_1 - k_{d1}\dot{q}_1(t-h) + \frac{1}{2}\phi_1(q)\hat{\theta} + \frac{1}{2}\phi_1(q)\hat{\theta}(t-h), \\
\tau_2(t, t-h) = m_2 l_{c2} g \cos(q_1 + q_2) - \lambda_2(q_2 - q_2^0) - k_{d22}\dot{q}_2 - k_{d2}\dot{q}_2(t-h) \\
\qquad\qquad + \frac{1}{2}\phi_2(q)\hat{\theta} + \frac{1}{2}\phi_2(q)\hat{\theta}(t-h), \\
\dot{\hat{\theta}} = -\frac{1}{2}\Gamma_0^{-1}\Phi^{\mathsf{T}}(q)\dot{q} - \frac{1}{2}\Gamma_0^{-1}\Phi^{\mathsf{T}}(q)\dot{q}(t-h), \\
\hat{\theta}(t) = \varphi(t), \quad t \in [-h, 0].
\end{cases}
\tag{5.10}
$$

In order to show the effectiveness of the controller (5.10), simulation is investigated for system (5.1) whose physical parameters are the same as those in [7]. The target point $q^0 = [1.57, 0]$ is considered. In order to test the robustness of controller, square disturbances of amplitude $[8, 6]^T$ are added to the system during 1.0s-1.5s. Two different points: $X_0^1 = [1.57, 0, 0, 0]^T$, $X_0^2 = [2.0, 0.5, 0, 0]^T$ are considered as the initial points of system (5.1).

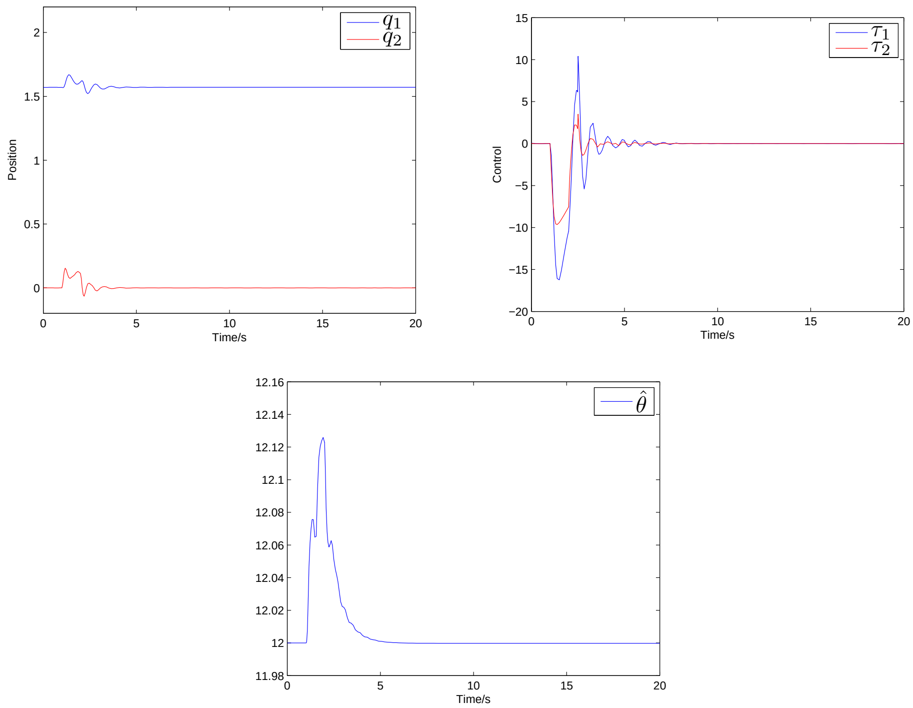

Figure 2: Responses of position q, control τ, estimate $\hat{\theta}$.

Figure 2 is the response of the system with the time delay $h = 0.5s$ and the initial point X_0^1 which is the same as the target point. Figure 2 shows the convergences of position q, the control signal τ used here and the estimate $\hat{\theta}$ of the payload m_p, respectively. The same disturbances of amplitude are chosen in [22]. Although the time delays are considered in the same system, we can obtain a good effect of convergence under the proposed controller (5.10) as well as in [22]. It illustrates that the designed controller has well robust properties when it is subject to external disturbances and time delays.

Figure 3 illustrates the responses of the system with the different time delays. The left column subgraphs of Figure 3 are the responses of the system with the time delay $h = 0.5s$ and the initial point X_0^2 which is different from the target point. It indicates the position q can converge to target point when initial point is different from target point which illustrates the effectiveness of the controller (5.10). The right column subgraphs of Figure 3 are the responses of the system with the time delay $h = 2.5s$. We also choose initial point X_0^2. From Figure 3, we can see that the proposed controller works well even in the presence of different delays. But it needs taking much more time when time delays are bigger.

The above simulation results demonstrate that the energy-based robust adaptive controller (5.10) is effective for the control of position, and for dealing with both unknown parameters and external disturbances. Furthermore, the well convergence of the controller τ can give us an evidence that it is effective for the control of the delayed robot manipulator (2.2).

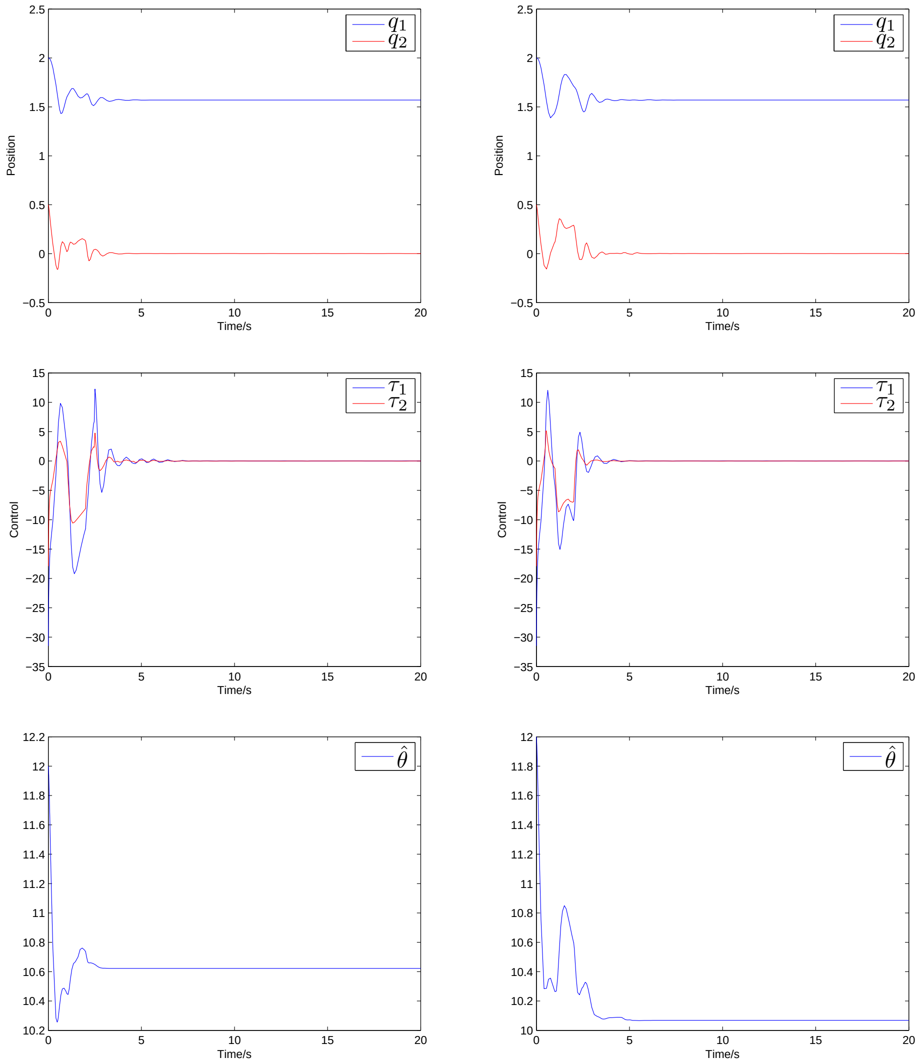

Figure 3: Responses of position q, control τ, estimate $\hat{\theta}$.

6 Conclusions

In this paper, we have proposed an energy-based approach for $\mathcal{L}_2$-disturbance attenuation control of uncertain robot manipulator in the presence of time delay. A new delayed Hamiltonian formulation has been given out both in the condition of fully actuated and underactuated uncertain robot manipulator with time delay. Based on the obtained delayed Hamiltonian formulation, an energy-based adaptive $\mathcal{L}_2$-disturbance attenuation controller has been proposed for the system. Simulation has shown the effectiveness of the controller in handing disturbance and unknown parameters in delayed robot manipulator. It is noteworthy for further study that the proposed energy-based control method for robot manipulator with time delay can be used in other robot dynamics in the presence of delay. However, the crucial question depends on the suitable delayed Hamiltonian formulation of a robot system.

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Competing Interests

The authors declare that no competing interests exist.

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