



Minimum Transversal Eccentric Dominating Energy of Graphs

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

Article Information

DOI: 10.9734/ARJOM/2023/v19i10741

Open Peer Review History:

This journal follows the Advanced Open Peer Review policy. Identity of the Reviewers, Editor(s) and additional Reviewers, peer review comments, different versions of the manuscript, comments of the editors, etc are available here:
<https://www.sdiarticle5.com/review-history/105802>

Original Research Article

Received: 04/07/2023
Accepted: 10/09/2023
Published: 19/09/2023

Abstract

For a graph G , the minimum transversal eccentric dominating energy $\mathbb{E}_{ted}(G)$ is the sum of the eigenvalues obtained from the minimum transversal eccentric dominating $n \times n$ matrix $\mathbb{M}_{ted}(G) = (m_{ij})$. In this paper $\mathbb{E}_{ted}(G)$ of some standard graphs are computed. Properties, upper and lower bounds for $\mathbb{E}_{ted}(G)$ are established.

Keywords: Transversal domination; eccentricity; transversal eccentric domination number; minimum transversal eccentric dominating set; eigenvalues, energy.

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1 Introduction

The concept of ‘energy of a graph’ was introduced by I. Gutman [1] in 1978. M.R. Rajesh Kanna et al [2] found the minimum dominating energy of a graph. Tejaskumar R, A Mohamed Ismayil and Ivan Gutman [3] introduced ‘minimum eccentric dominating energy of graphs’. For a graph $G = (V, E)$, a set $S \subseteq V$ is said to be a dominating set, if every vertex in $V - S$ is adjacent to some vertex in S . The concept of ‘eccentric domination’ was introduced by T.N. Janakiraman et al. [4] in 2010. The eccentricity $e(v)$ of v is the distance to a vertex farthest from v . Thus, $e(v) = \max \{d(u, v): u \in V\}$. For a vertex v , each vertex at a distance $e(v)$ from v is an eccentric vertex. Eccentric set of a vertex v is defined as $E(v) = \{u \in V(G): d(u, v) = e(v)\}$. A set $D \subseteq V(G)$ is an eccentric dominating set if D is a dominating set of G and for every $v \in V - D$, there exists at least one eccentric vertex of v in D . Nayaka S.R et al. [5] introduced transversal domination in graphs. Riyaz Ur Rehman A and A Mohamed Ismayil [6] introduced transversal eccentric domination in graphs. A eccentric dominating set S is called a transversal eccentric dominating set if it intersects every minimum eccentric dominating set D' i.e. $S \cap D' \neq \emptyset$. Inspired by Tejaskumar et al [3] we introduce minimum transversal eccentric dominating energy $\mathbb{E}_{ted}(G)$ of graphs. In this paper we discuss some properties on $\mathbb{E}_{ted}(G)$ of standard graphs.

2 The Minimum Transversal Eccentric Dominating Energy- $\mathbb{E}_{ted}(G)$

Definition 2.1: Let $G = (V, E)$ be a simple graph where $V(G) = \{v_1, v_2, \dots, v_n\}$ where $n \in \mathbb{N}$ is the set of vertices and E is the set of edges. Let D be a minimum transversal eccentric dominating set of G then the minimum transversal eccentric dominating matrix of G is a $n \times n$ matrix defined by $\mathbb{M}_{ted}(G) = (m_{ij})$, where

$$(m_{ij}) = \begin{cases} 1, \text{ if } v_i \text{ or } v_j \cap D \neq \emptyset \text{ and } v_i \in E(v_j) \text{ or } v_j \in E(v_i), \\ 1, \text{ if } i = j \text{ and } v_i \in D, \\ 0, \text{ otherwise} \end{cases}$$

Definition 2.2: The characteristic polynomial of $\mathbb{M}_{ted}(G)$ is denoted by $\mathcal{P}_n(G, \alpha) = \det (\mathbb{M}_{ted}(G) - \alpha I)$.

Definition 2.3: The minimum transversal eccentric dominating eigenvalues of G are the eigenvalues of minimum transversal eccentric dominating matrix $\mathbb{M}_{ted}(G)$. Since $\mathbb{M}_{ted}(G)$ is symmetric and real. We label the eigen values in non-increasing order $\alpha_1 \geq \alpha_2 \geq \dots \geq \alpha_n$.

Definition 2.4: The minimum transversal eccentric dominating energy of G is defined by $\mathbb{E}_{ted}(G) = \sum_{i=1}^n |\alpha_i|$.

Remark 2.1: The trace of $\mathbb{M}_{ted}(G)$ =Transversal eccentric domination number.

Example 2.1:

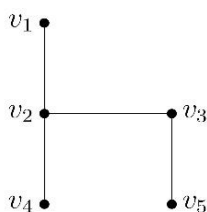


Fig 1. Fork graph

Table 1. The vertex with Eccentricity $e(v)$ and Eccentric vertex $E(v)$

Vertex	Eccentricity $e(v)$	Eccentric vertex $E(v)$
v_1	3	v_5
v_2	2	v_5
v_3	2	v_1, v_4
v_4	3	v_5
v_5	3	v_1, v_4

The minimum transversal eccentric dominating sets of fork graph are $D_1 = \{v_1, v_2, v_5\}$, $D_2 = \{v_1, v_4, v_5\}$, $D_3 = \{v_2, v_3, v_5\}$ and $D_4 = \{v_2, v_4, v_5\}$.

1. $D_1 = \{v_1, v_2, v_5\}$,

$$M_{ted}(G) = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 \end{pmatrix}$$

The characteristic polynomial $\mathcal{P}_n(G, \alpha) = -\alpha^5 + 3\alpha^4 + \alpha^3 - 5\alpha + 1$.

Minimum transversal eccentric dominating eigenvalues are $\alpha_1 \approx 2.705$, $\alpha_2 \approx 1.3835$, $\alpha_3 \approx 0.5102$, $\alpha_4 \approx -0.4599$, $\alpha_5 \approx -1.1388$.

Minimum transversal eccentric dominating energy $\mathbb{E}_{ted}(G) \approx 6.1974$.

2. $D_2 = \{v_1, v_4, v_5\}$,

$$M_{ted}(G) = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \end{pmatrix}$$

The characteristic polynomial $\mathcal{P}_n(G, \alpha) = -\alpha^5 + 3\alpha^4 + \alpha^3 - 5\alpha + 1$.

Minimum transversal eccentric dominating eigenvalues are $\alpha_1 \approx 2.705$, $\alpha_2 \approx 1.3835$, $\alpha_3 \approx 0.5102$, $\alpha_4 \approx -0.4599$, $\alpha_5 \approx -1.1388$.

Minimum transversal eccentric dominating energy $\mathbb{E}_{ted}(G) \approx 6.1974$.

3. $D_3 = \{v_2, v_3, v_5\}$,

$$M_{ted}(G) = \begin{pmatrix} 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 \end{pmatrix}$$

The characteristic polynomial $\mathcal{P}_n(G, \alpha) = -\alpha^5 + 3\alpha^4 + \alpha^3 - 6\alpha^2 + \alpha + 1$.

Minimum transversal eccentric dominating eigenvalues are $\alpha_1 \approx 2.5563$, $\alpha_2 \approx 1.5063$, $\alpha_3 \approx 0.5896$, $\alpha_4 \approx -0.3342$, $\alpha_5 \approx -1.138$.

Minimum transversal eccentric dominating energy $\mathbb{E}_{ted}(G) \approx 6.3044$.

4. $D_4 = \{v_2, v_4, v_5\}$,

$$M_{ted}(G) = \begin{pmatrix} 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \end{pmatrix}$$

The characteristic polynomial $\mathcal{P}_n(G, \alpha) = -\alpha^5 + 3\alpha^4 + \alpha^3 - 6\alpha^2 + \alpha + 1$.

Minimum transversal eccentric dominating eigenvalues are $\alpha_1 \approx 2.5563$, $\alpha_2 \approx 1.5063$, $\alpha_3 \approx 0.5896$, $\alpha_4 \approx -0.3342$, $\alpha_5 \approx -1.138$.

Minimum transversal eccentric dominating energy $\mathbb{E}_{ted}(G) \approx 6.3044$.

Remark 2.2: The minimum transversal eccentric dominating energy depends on the minimum transversal eccentric dominating set.

3 Minimum Transversal Eccentric Dominating Energy of some Standard Graphs

In this section we find the $\mathbb{E}_{ted}(G)$ of complete graph, cocktail party and crown graph.

Theorem 3.1: For a complete graph K_n where $n > 2$ the minimum transversal eccentric dominating energy of a complete graph is $\mathbb{E}_{ted}(K_n) = n$.

Proof: Let K_n be a complete graph with the vertex set $V = \{v_1, v_2, \dots, v_n\}$. The minimum transversal eccentric dominating set is $D = \{v_1\}$, then

$$\mathbb{M}_{ted}(K_n) = \begin{pmatrix} 1 & 1 & 1 & \dots & 1 & 1 & 1 \\ 1 & 0 & 1 & \dots & 1 & 1 & 1 \\ 1 & 1 & 0 & \dots & 1 & 1 & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 1 & 1 & 1 & \dots & 0 & 1 & 1 \\ 1 & 1 & 1 & \dots & 1 & 0 & 1 \\ 1 & 1 & 1 & \dots & 1 & 1 & 0 \end{pmatrix}_{n \times n}$$

Characteristic polynomial is $\mathcal{P}_n(K_n, \alpha) = \det(\mathbb{M}_{ted}(K_n) - \alpha I)$.

$$= \begin{vmatrix} 1 - \psi & 1 & 1 & \dots & 1 & 1 & 1 \\ 1 & -\psi & 1 & \dots & 1 & 1 & 1 \\ 1 & 1 & -\psi & \dots & 1 & 1 & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 1 & 1 & 1 & \dots & -\psi & 1 & 1 \\ 1 & 1 & 1 & \dots & 1 & -\psi & 1 \\ 1 & 1 & 1 & \dots & 1 & 1 & -\psi \end{vmatrix}$$

The characteristic equation is $\mathcal{P}_n(K_n, \alpha) = (-1)^n \alpha^n - (-1)^n n \alpha^{n-1}$.

The minimum transversal eccentric dominating eigenvalues are

$$\alpha = 0, \\ \alpha = n.$$

The minimum transversal eccentric dominating energy of the complete graph K_n is given by $\mathbb{E}_{ted}(K_n) = n$.

Theorem 3.2: For a cocktail party where $n \geq 4$ and crown graph G where $n \geq 6$ the minimum transversal eccentric dominating energy

$$\mathbb{E}_{ted}(G) = 2 + \left| \frac{1+\sqrt{5}}{2} \right| \left[\binom{n}{2} - 1 \right] + \left| \frac{1-\sqrt{5}}{2} \right| \left[\binom{n}{2} - 1 \right].$$

Proof: Let G be a graph with the vertex set $V = \{v_1, v_2, \dots, v_n\}$. The minimum transversal eccentric dominating set is $D = \{v_1, v_2, \dots, v_{\frac{n}{2}}\}$, $|D| = n/2$ then

$$\mathbb{M}_{ted}(G) = \begin{pmatrix} 1 & 0 & 0 & 0 & & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & \dots & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & \dots & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & & 0 & 1 & 0 & 0 \\ & & \vdots & & \ddots & & \vdots & & \\ 0 & 0 & 1 & 0 & & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \dots & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & & 0 & 0 & 0 & 0 \end{pmatrix}_{n \times n}$$

Characteristic polynomial is $\mathcal{P}_n(G, \alpha) = \det(\mathbb{M}_{ted}(G) - \alpha I)$.

$$= \begin{vmatrix} 1-\alpha & 0 & 0 & 0 & & 0 & 0 & 1 & 0 \\ 0 & 1-\alpha & 0 & 0 & \dots & 0 & 0 & 0 & 1 \\ 0 & 0 & 1-\alpha & 0 & \dots & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1-\alpha & & 0 & 1 & 0 & 0 \\ & & \vdots & & \ddots & & \vdots & & \\ & 0 & 0 & 1 & 0 & & 1-\alpha & 0 & 0 & 0 \\ & 0 & 0 & 0 & 1 & \dots & 0 & -\alpha & 0 & 0 \\ & 1 & 0 & 0 & 0 & & 0 & 0 & -\alpha & 0 \\ & 0 & 1 & 0 & 0 & & 0 & 0 & 0 & -\alpha \end{vmatrix}$$

The characteristic equation is $\mathcal{P}_n(G, \alpha) = \alpha(\alpha - 2)(\alpha^2 - \alpha - 1)^{\frac{n-2}{2}}$.

The minimum transversal eccentric dominating eigenvalues are

$$\begin{aligned} \alpha &= 0, \\ \alpha &= 2, \\ \alpha &= \frac{1 + \sqrt{5}}{2} \left[\left(\frac{n}{2} - 1\right) \text{ times} \right], \\ \alpha &= \frac{1 - \sqrt{5}}{2} \left[\left(\frac{n}{2} - 1\right) \text{ times} \right], \end{aligned}$$

The minimum transversal eccentric dominating energy of the cocktail party and crown graph G is given by

$$\begin{aligned} \mathbb{E}_{ted}(G) &= 0 + 2 + \frac{1+\sqrt{5}}{2} \left[\left(\frac{n}{2} - 1\right) \right] + \frac{1-\sqrt{5}}{2} \left[\left(\frac{n}{2} - 1\right) \right]. \\ \mathbb{E}_{ted}(G) &= 2 + \frac{1+\sqrt{5}}{2} \left[\left(\frac{n}{2} - 1\right) \right] + \frac{1-\sqrt{5}}{2} \left[\left(\frac{n}{2} - 1\right) \right]. \end{aligned}$$

4 Properties of Minimum Transversal Eccentric Dominating Eigenvalues

In this section we discuss the properties of eigenvalues of $\mathbb{M}_{ted}(G)$ for complete, crown and cocktail party graphs.

Theorem 4.1: Let D be a minimum transversal eccentric dominating set and $\alpha_1, \alpha_2, \dots, \alpha_n$ are the eigenvalues of minimum transversal eccentric dominating matrix $\mathbb{M}_{ted}(G)$ then

1. For any graph G , $\sum_{i=1}^n \alpha_i = |D|$,
2. For a complete graph K_n where $n > 2$, $\sum_{i=1}^n \alpha_i^2 = |D| + (n)(n - 1)$,
3. For a crown and cocktail party graph G then $\sum_{i=1}^n \alpha_i^2 = |D| + n$.

Proof:

1. We know that the sum of eigenvalues of $\mathbb{M}_{ted}(G)$ is the trace of $\mathbb{M}_{ted}(G)$.

$$\sum_{i=1}^n \alpha_i = \sum_{i=1}^n m_{ii} = |D|.$$

2. Similarly, for a complete graph K_n sum of square of eigen values of $\mathbb{M}_{ted}(G)$ is trace of $[\mathbb{M}_{ted}(G)]^2$

$$\begin{aligned} \sum_{i=1}^n \alpha_i^2 &= \sum_{i=1}^n \sum_{j=1}^n m_{ij} m_{ij} \\ \sum_{i=1}^n \alpha_i^2 &= \sum_{i=1}^n (m_{ii})^2 + \sum_{i \neq j} m_{ij} m_{ij} \\ \sum_{i=1}^n \alpha_i^2 &= \sum_{i=1}^n (m_{ii})^2 + 2 \sum_{i < j} (m_{ij})^2 \\ \sum_{i=1}^n \alpha_i^2 &= |D| + (n)(n - 1) \end{aligned}$$

Since for a complete graph K_n , $2 \sum_{i < j} (m_{ij})^2 = (n)(n - 1)$.

3. Similarly, for a crown and cocktail party graph G sum of square of eigenvalues of $\mathbb{M}_{ted}(G)$ is trace of $[\mathbb{M}_{ted}(G)]^2$

$$\begin{aligned} \sum_{i=1}^n \alpha_i^2 &= \sum_{i=1}^n \sum_{j=1}^n m_{ij} m_{ij} \\ \sum_{i=1}^n \alpha_i^2 &= \sum_{i=1}^n (m_{ii})^2 + \sum_{i \neq j} m_{ij} m_{ij} \\ \sum_{i=1}^n \alpha_i^2 &= \sum_{i=1}^n (m_{ii})^2 + 2 \sum_{i < j} (m_{ij})^2 \\ \sum_{i=1}^n \alpha_i^2 &= |D| + n \end{aligned}$$

Since for a crown and cocktail party graph G , $2 \sum_{i < j} (m_{ij})^2 = n$.

5 Bounds for Minimum Transversal Eccentric Dominating Energy

Similarly to McClelland’s [7] bounds for energy of a graph, bounds for $\mathbb{E}_{ted}(G)$ are given in this section.

Theorem 5.1: For a complete graph K_n where $n > 1$, if D be the minimum transversal eccentric dominating set and $W = |\det \mathbb{M}_{ted}(G)|$ then

$$\sqrt{|D| + n(n - 1) + n(n - 1)W^{2/n}} \leq \mathbb{E}_{ted}(K_n) \leq \sqrt{n(n(n - 1) + |D|)}.$$

Proof: By Cauchy schwarz inequality $(\sum_{i=1}^n g_i h_i)^2 \leq (\sum_{i=1}^n g_i^2)(\sum_{i=1}^n h_i^2)$. If $g_i = 1$ and $h_i = \alpha_i$ then

$$\left(\sum_{i=1}^n |\alpha_i| \right)^2 \leq \left(\sum_{i=1}^n 1 \right) \left(\sum_{i=1}^n \alpha_i^2 \right)$$

$$\begin{aligned} (\mathbb{E}_{ted}(G))^2 &\leq n(|D| + n(n - 1)) \\ \Rightarrow \mathbb{E}_{ted}(G) &\leq \sqrt{n(|D| + n(n - 1))} \end{aligned}$$

Since the arithmetic mean is not smaller than geometric mean we have

$$\begin{aligned} \frac{1}{n(n-1)} \sum_{i \neq j} |\alpha_i| |\alpha_j| &\geq \left[\prod_{i \neq j} |\alpha_i| |\alpha_j| \right]^{\frac{1}{n(n-1)}} \\ \frac{1}{n(n-1)} \sum_{i \neq j} |\alpha_i| |\alpha_j| &= \left[\prod_{i=1}^n |\alpha_i|^{2(n-1)} \right]^{\frac{1}{n(n-1)}} \\ \frac{1}{n(n-1)} \sum_{i \neq j} |\alpha_i| |\alpha_j| &= \left[\prod_{i=1}^n |\alpha_i| \right]^{\frac{2}{n}} \\ \frac{1}{n(n-1)} \sum_{i \neq j} |\alpha_i| |\alpha_j| &= \left[\prod_{i=1}^n \alpha_i \right]^{\frac{2}{n}} \\ \frac{1}{n(n-1)} \sum_{i \neq j} |\alpha_i| |\alpha_j| &= |\det \mathbb{M}_{ted}(G)|^{\frac{2}{n}} = W^{\frac{2}{n}} \\ \sum_{i \neq j} |\alpha_i| |\alpha_j| &\geq n(n-1)W^{\frac{2}{n}} \end{aligned}$$

Now consider

$$\begin{aligned} (\mathbb{E}_{ted}(G))^2 &= \left(\sum_{i=1}^n |\alpha_i| \right)^2 \\ (\mathbb{E}_{ted}(G))^2 &= \left(\sum_{i=1}^n |\alpha_i| \right)^2 + \sum_{i \neq j} |\alpha_i| |\alpha_j| \\ (\mathbb{E}_{ted}(G))^2 &= (|D| + n(n-1)) + n(n-1)W^{\frac{2}{n}} \\ \mathbb{E}_{ted}(G) &\geq \sqrt{(|D| + n(n-1)) + n(n-1)W^{\frac{2}{n}}} \end{aligned}$$

Theorem 5.2: For a crown graph where $n \geq 6$ and cocktail party graph G where $n \geq 4$, be the minimum transversal eccentric dominating set and $W = |\det \mathbb{M}_{ted}(G)|$ then

$$\sqrt{|D| + n + n(n-1)W^{\frac{2}{n}}} \leq \mathbb{E}_{ted}(G) \leq \sqrt{n(n + |D|)}.$$

Proof: The proof follows on the similar lines of theorem-5.1.

Theorem 5.3: If $\alpha_1(G)$ is the largest minimum transversal eccentric dominating eigenvalue of $\mathbb{M}_{ted}(G)$ then

1. For a complete graph K_n , $\alpha_1(G) \geq \frac{|D|+n(n-1)}{n}$,
2. For a crown and cocktail party graph G , $\alpha_1(G) \geq \frac{|D|+n}{n}$.

Proof:

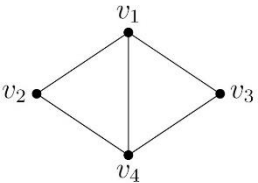
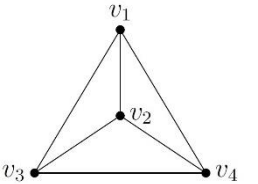
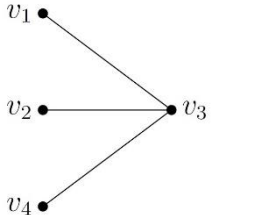
1. Let Y be a non-zero vector, then by ref. [8], we have $\alpha_1(\mathbb{M}_{ted}(G)) = \max_{Y \neq 0} \frac{Y^T \mathbb{M}_{ted}(G) Y}{Y^T Y}$.

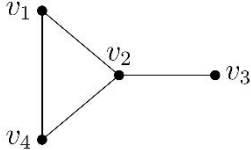
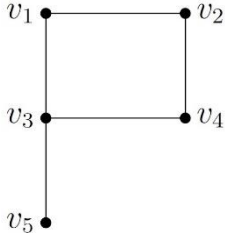
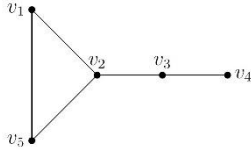
$$\alpha_1(\mathbb{M}_{ted}(G)) \geq \frac{U^T \mathbb{M}_{ted}(G) U}{U^T U} = \frac{|D|+n(n-1)}{n} \text{ where } U \text{ is the unit matrix.}$$

2. Let Y be a non-zero vector, then by ref. [8], we have $\mu_1(\mathbb{M}_{ted}(G)) = \max_{Y \neq 0} \frac{Y^T \mathbb{M}_{ted}(G) Y}{Y^T Y}$.

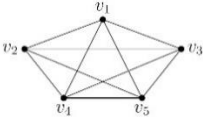
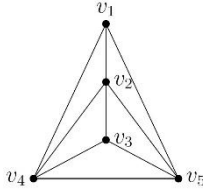
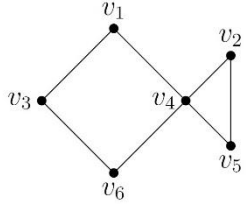
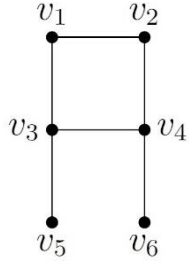
$$\mu_1(\mathbb{M}_{ted}(G)) \geq \frac{U^T \mathbb{M}_{ted}(G) U}{U^T U} = \frac{|D|+n}{n} \text{ where } U \text{ is the unit matrix [9-13].}$$

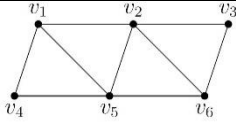
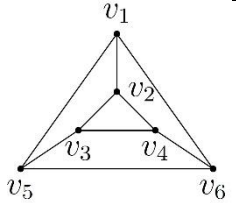
Table 2. Characteristic equation $\mathcal{P}_n(G, \alpha)$, Roots $\alpha(G)$ and Energy $\mathbb{E}_{ted}(G)$ of Minimum TED sets of various standard graphs are tabulated

Graph	Figure	Minimum TED set	Characteristic equation $\mathcal{P}_n(G, \alpha)$	Roots $\alpha(G)$	Energy $\mathbb{E}_{ted}(G)$
Diamond graph		$\{v_2, v_3\}$,	$\alpha^4 - 2\alpha^3 - 4\alpha^2$.	$\alpha_1 = 3.2361$, $\alpha_2 = 0$, $\alpha_3 = -1.2361$.	4
Tetrahedral graph		$\{v_1, v_2, v_3, v_4\}$,	$\alpha^4 - 2\alpha^3$.	$\alpha_1 = 3.2361$, $\alpha_2 = 0$.	4
Claw graph		$\{v_1, v_3\}$,	$\alpha^4 - 2\alpha^3 - 4\alpha^2$,	$\alpha_1 = 3.2361$, $\alpha_2 = 0$, $\alpha_3 = -1.2361$.	4.4722
		$\{v_2, v_3\}$,	$\alpha^4 - 2\alpha^3 - 4\alpha^2 + \alpha + 1$,	$\alpha_1 = 3.1401$, $\alpha_2 = 0.5712$, $\alpha_3 = -0.4378$, $\alpha_4 = -1.2735$.	5.3866
		$\{v_3, v_4\}$.	$\alpha^4 - 2\alpha^3 - 4\alpha^2 + \alpha + 1$.	$\alpha_1 = 3.1401$, $\alpha_2 = 0.5712$, $\alpha_3 = -0.4378$, $\alpha_4 = -1.2735$.	5.3866

Graph	Figure	Minimum TED set	Characteristic equation $\mathcal{P}_n(G, \alpha)$	Roots $\alpha(G)$	Energy $\mathbb{E}_{ted}(G)$
Paw graph		$\{v_1, v_3\}$,	$\alpha^4 - 2\alpha^3 - 3\alpha^2 + \alpha + 1$,	$\alpha_1 = 2.8794$, $\alpha_2 = 0.6527$, $\alpha_3 = -0.5321$, $\alpha_4 = -1$.	5.0642
		$\{v_2, v_3\}$,	$\alpha^4 - 2\alpha^3 - 3\alpha^2 + \alpha + 1$,	$\alpha_1 = 2.8794$, $\alpha_2 = 0.6527$, $\alpha_3 = -0.5321$, $\alpha_4 = -1$.	5.0642
		$\{v_3, v_4\}$.	$\alpha^4 - 2\alpha^3 - 3\alpha^2 + 2\alpha + 2$.	$\alpha_1 = 2.7321$, $\alpha_2 = 1$, $\alpha_3 = -0.7321$, $\alpha_4 = -1$.	5.4642
Banner graph		$\{v_2, v_5\}$.	$-\alpha^5 + 2\alpha^4 + 3\alpha^3 - 3\alpha^2 - 2\alpha$.	$\alpha_1 = 2.5962$, $\alpha_2 = 1.1826$, $\alpha_3 = 0$, $\alpha_4 = -0.5157$, $\alpha_5 = -1.2631$.	5.5576
(3,2) Tadpole graph		$\{v_1, v_4\}$,	$-\alpha^5 + 2\alpha^4 + 3\alpha^3 - 3\alpha^2 - 2\alpha$,	$\alpha_1 = 2.5962$, $\alpha_2 = 1.1826$, $\alpha_3 = 0$, $\alpha_4 = -0.5157$, $\alpha_5 = -1.2631$.	5.5576
		$\{v_4, v_5\}$.	$-\alpha^5 + 2\alpha^4 + 3\alpha^3 - 4\alpha^2 - \alpha + 1$.		6.0164

Graph	Figure	Minimum TED set	Characteristic equation $\mathcal{P}_n(G, \alpha)$	Roots $\alpha(G)$	Energy $\mathbb{E}_{ted}(G)$
				$\alpha_1 = 2.5231,$ $\alpha_2 = 1,$ $\alpha_3 = 0.4851,$ $\alpha_4 = -0.5669,$ $\alpha_5 = -1.4413.$	
Kite graph		$\{v_2, v_4\}.$	$-\alpha^5 + 2\alpha^4 + 3\alpha^3$ $- 3\alpha^2$ $- 2\alpha.$	$\alpha_1 = 2.5962,$ $\alpha_2 = 1.1826,$ $\alpha_3 = 0,$ $\alpha_4 = -0.5157,$ $\alpha_5 = -1.2631.$	5.5576
Dart graph		$\{v_2, v_3\},$	$-\alpha^5 + 2\alpha^4 + 6\alpha^3,$	$\alpha_1 = 3.6458,$ $\alpha_2 = 0,$ $\alpha_3 = -1.6458.$	5.2916
		$\{v_2, v_4\}.$	$-\alpha^5 + 2\alpha^4 + 6\alpha^3$ $- 2\alpha^2$ $- 2\alpha.$	$\alpha_1 = 3.6458,$ $\alpha_2 = 0.7018,$ $\alpha_3 = 0,$ $\alpha_4 = -0.4685,$ $\alpha_5 = -1.7365.$	6.41
Cricket graph		$\{v_3, v_4\},$	$-\alpha^5 + 2\alpha^4 + 6\alpha^3,$	$\alpha_1 = 3.6458,$ $\alpha_2 = 0,$ $\alpha_3 = -1.6458.$	5.2916
		$\{v_4, v_5\}.$	$-\alpha^5 + 2\alpha^4 + 6\alpha^3$ $- 2\alpha^2$ $- 2\alpha.$	$\alpha_1 = 3.5032,$ $\alpha_2 = 0.7018,$ $\alpha_3 = 0,$ $\alpha_4 = -0.4685,$ $\alpha_5 = -1.7365.$	6.41

Graph	Figure	Minimum TED set	Characteristic equation $\mathcal{P}_n(G, \alpha)$	Roots $\alpha(G)$	Energy $\mathbb{E}_{ted}(G)$
Pentatope graph		$\{v_1, v_2, v_3, v_4, v_5\}$.	$-\alpha^5 + 5\alpha^4$.	$\alpha_1 = 5, \quad \alpha_2 = 0.$	5
Johnson solid skeleton 12 graph		$\{v_1, v_3\}$.	$-\alpha^5 + 2\alpha^4 + 6\alpha^3$.	$\alpha_1 = 3.6458,$ $\alpha_2 = 0,$ $\alpha_3 = -1.6458.$	5.2916
Fish graph		$\{v_2, v_3\}$,	$\alpha^6 - 2\alpha^5 - 4\alpha^4 + 4\alpha^3$ $+ 4\alpha^2,$	$\alpha_1 = 2.7321,$ $\alpha_2 = 1.4142,$ $\alpha_3 = 0,$ $\alpha_4 = -0.7321,$ $\alpha_5 = -1.4142.$	6.2926
		$\{v_3, v_5\}$.	$\alpha^6 - 2\alpha^5 - 4\alpha^4 + 6\alpha^3$ $+ 2\alpha^2$ $- 2\alpha.$	$\alpha_1 = 2.5772,$ $\alpha_2 = 1.292,$ $\alpha_3 = 0.522,$ $\alpha_4 = 0,$ $\alpha_5 = -0.6677,$ $\alpha_6 = -1.7235.$	6.7824
A graph		$\{v_1, v_5, v_6\}$,	$\alpha^6 - 3\alpha^5 - 3\alpha^4 + 9\alpha^3$ $+ \alpha^2$ $- 4\alpha$ $- 1,$	$\alpha_1 = 3.0233,$ $\alpha_2 = 1.3883,$ $\alpha_3 = 1,$ $\alpha_4 = -0.2853,$ $\alpha_5 = -0.5198,$ $\alpha_6 = -1.6065.$	6.823
		$\{v_2, v_5, v_6\}$.	$\alpha^6 - 3\alpha^5 - 3\alpha^4 + 10\alpha^3$ $- \alpha^2$ $- 3\alpha.$		7.5092

Graph	Figure	Minimum TED set	Characteristic equation $\mathcal{P}_n(G, \alpha)$	Roots $\alpha(G)$	Energy $E_{ted}(G)$
				$\alpha_1 = 2.9446,$ $\alpha_2 = 1.541,$ $\alpha_3 = 0.769,$ $\alpha_4 = 0,$ $\alpha_5 = -0.4862,$ $\alpha_6 = -1.769.$	
4-polynomial graph		$\{v_3, v_4\}.$	$\alpha^6 - 2\alpha^5 - 4\alpha^4 + 4\alpha^3$ $+ 4\alpha^2.$	$\alpha_1 = 2.7321,$ $\alpha_2 = 1.4142,$ $\alpha_3 = 0,$ $\alpha_4 = -0.7321,$ $\alpha_5 = -1.4142.$	6.2926
3-prism graph		$\{v_1, v_5, v_6\},$	$\alpha^6 - 3\alpha^5 - 3\alpha^4 + 11\alpha^3$ $+ 3\alpha^2$ $- 9\alpha$ $- 4,$	$\alpha_1 = 2.5616,$ $\alpha_2 = 1.618,$ $\alpha_3 = 1.618,$ $\alpha_4 = -0.618,$ $\alpha_5 = -0.618,$ $\alpha_6 = -1.5616.$	8.5952
		$\{v_2, v_3, v_4\}.$	$\alpha^6 - 3\alpha^5 - 3\alpha^4 + 11\alpha^3$ $+ 3\alpha^2$ $- 9\alpha$ $- 4.$	$\alpha_1 = 2.5616,$ $\alpha_2 = 1.618,$ $\alpha_3 = 1.618,$ $\alpha_4 = -0.618,$ $\alpha_5 = -0.618,$ $\alpha_6 = -1.5616.$	8.5952

6 Conclusion

In this paper minimum transversal eccentric dominating energy of a graph is introduced. The transversal eccentric dominating energy of some standard graphs are calculated. Results related to the upper and lower bounds of the energy of standard graphs are stated and proved.

Competing Interests

Authors have declared that no competing interests exist.

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