Utilizing ISO-Value Field Curves in Lieu of Magnetic Field Lines Amid Infinite and Parallel Electrical Wires

Geoffroy Auvert

INPG, Grenoble Alpes University, Saint Martin d'Hères, France
Email: geoffroy.auvert@grenoble-inp.org

Abstract

Building on a new model proposed recently for calculating constant electro-magnetic field values, the present article explores the electro-magnetic field configuration generated by parallel electrical wires. This imposes a reevaluation of the drawing procedure for constructing field curves with a constant field values around multiple parallel electrical conducting wires. To achieve this, we employ methods akin to those used for creating contours on topographical maps, ensuring a consistent numerical field value along the entire length of the field curves. Subsequent calculations will be conducted for scenarios where wires are not parallel.

Keywords

Specific Field Value, Parallel Electrical Wires, Magnetic Field Vector, Field around Parallel Wires, Topographic Level Map

1. Introduction

In 1820, Oersted made observations that an electrical current flowing through a lengthy wire generates magnetic field vectors in its vicinity [1]. He defined this vector to be oriented perpendicular to the wire and contingent upon the electrical current. This vector has the capability to reorient small compasses and align iron filings [2]. Then, successive vectors can be drawn, forming curves traditionally referred to as magnetic field lines or magnetic flux lines [3]. These experimental lines resemble circles around an individual straight electrical wire, as reported by M. Zollner in 2002 [4]. Around this straight wire and employing the Biot-Savart law [5], these circles have been mathematically calculated [6]. In another scenario, such as when using a circular electrical wire, this vector can be observed by

employing iron filings [7]. It is feasible to compute this vector along the entire perpendicular axis of this current loop [6]. Along this axis, non-zero vectors have been computed [6], with their values diminishing to zero as they move further away from the circular wire. Since the first observation, no other concept has been developed to calculated magnetic field lines around electrical wires.

In our most recent publication regarding the field generated by an infinite electrical wire, we have introduced a new alternative definition for the field at a specific location in proximity to the wire [8]. We had suggested associating a single point with one physical field value and three vectors. In the present paper, we want to test this recently proposed model. To do so we will focus solely on the field value, without considering these three vectors. This consideration will reduce the number of parameters employed. As previously evaluating the magnitude of this field value, it will be directly proportional to the current in the wire and inversely proportional to the distance from it. This vector less definition will be employed throughout this article to determine the positions of points that share the same value generated by parallel electrical wires.

The employed method in this paper, is founded on the widely recognized process of constructing topographic contour maps, which comprises individual data points [9] [10]. In this topographic process, each point's value is assessed based on its elevation or depth and then positioned within horizontal planes [11]. Using this approach, field points sharing the same field value will be situated within a plane that is perpendicular to the wires.

In this paper, we depict points surrounding straight electrical-current wires with varying field values in several scenarios, including cases with different wire quantities: a single electrical-current wire, two wires with identical intensity and direction, parallel wires with opposing electrical-current directions, two parallel wires with different current intensities, scenarios requiring level assessment, and finally, instances involving at least five identical wires aligned in the same current direction. During the discussion, we elaborate on the validity of the superposition principle within our test of constant field values.

Units, used in this paper, are meters (m) for distances, Amperes (A) for electrical-current in wires, and Ampere/meter (A/m) for field values at a point placed everywhere out of the wires. To do so, the value of the vacuum permittivity is chosen equal to one.

In summary, this paper does not utilize classical magnetic field lines based on magnetic vectors. Instead, it employs as defined in our model previously published [8] of field values derived from physical calculations, enabling the construction of curves comprised of sequences of points.

2. Theoretical Bases for the First Application of Our Previously Published Model

This chapter uses a field definition with physical values as calculated in our previous article [8] knowing that this field definition has not been found in

other scientific publications. The main point was to introduce specific field values (not a vector) proportional to electrical current in wires and inversely to its distance.

The physical field value at any point on a perpendicular surface surrounding a solitary electrical wire can be assessed [6]. This value corresponds to I/r, where I represent the electrical current and r is the distance from the wire. In the case of three conducting wires, a single point "p" located within a plane perpendicular to the three wires derives its field value through the summation of these values, as depicted in Figure 1.

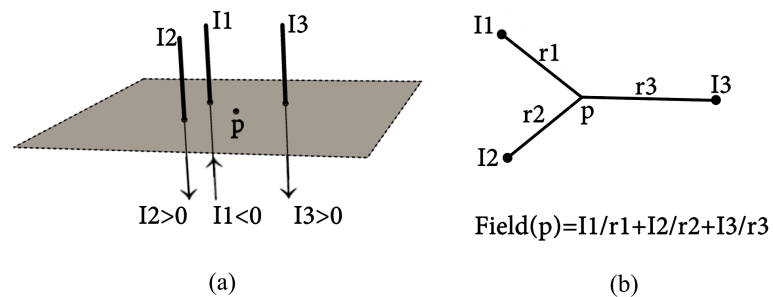$$\text{Field(p)}=I1/r1+I2/r2+I3/r3$$

(a)   (b)

Figure 1. Three infinite and parallel electrical-current wires going through a planar surface, imposing at point "p", a physical value of $\Sigma I/r$. In (a), the 3D view shows a planar surface and three parallel electrical wire in which currents direction are oriented up or down. In (b), top view of the planar surface gives the distance of point "p" to each wire. The field at that point is calculated depending on the three field values which are an addition of them.

In Figure 1(a) (left), when an electrical current flows downward, it imposes a positive field value of I/r with I > 0. In Figure 1(a) (left), when the electrical current is flowing upward, the field value becomes negative, denoted as I/r with I < 0. By superimposing all these field values at point "p," the result is equivalent to the summation of the three computed values, resulting in Σ I/r, as illustrated in Figure 1(b) (right). For the author, this holds significant physical significance as it imbues our model with a robust additivity property at all points within the plane perpendicular to the wires.

Note: The additivity of these values contradicts the conventional additivity of magnetic vectors.

3. Application

To generate a curve on the plane surface perpendicular to the wires, it is necessary for all points to share the same field value. (This type of curve does not resemble a traditional magnetic field line.) We will demonstrate that our drawing method bears a strong resemblance to creating contour lines on conventional topographic maps, where point values correspond to their elevation or depth [9].

3.1. One Electrical-Current Wire

Traditional magnetic field lines assume a circular shape when generated by a

single conducting wire [2]. These lines are determined using magnetic vectors equivalent to I/r. In Figure 2, our field value coincides with the magnetic vector because only one electrical wire is present. Each circular curve possesses a distinct "field value," which diminishes as the distance from the wire increases. In certain cases, field values may eventually reach zero as the distance approaches infinity.
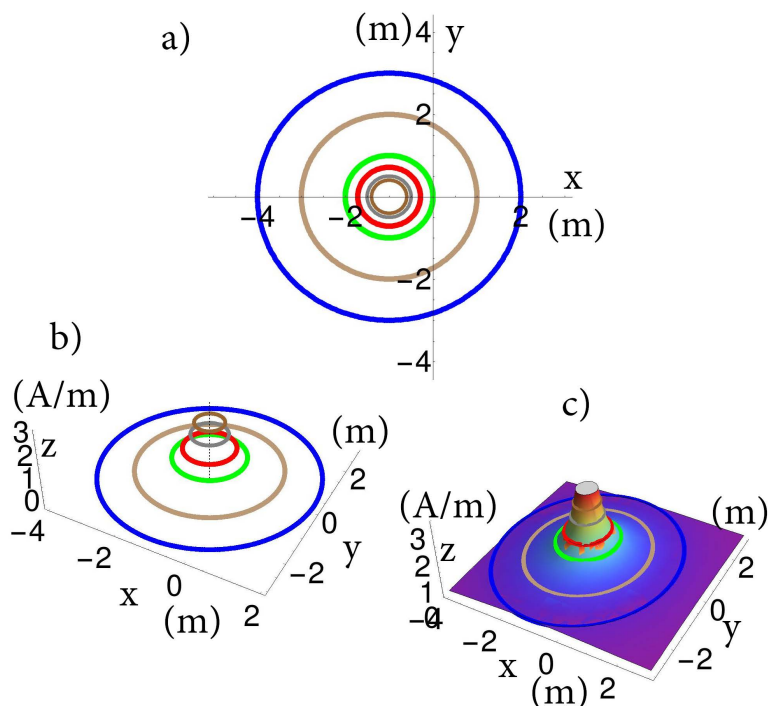

Figure 2. Representations with one 2D view and two 3D views, with one vertical infinite current wire of one (a) which is placed at position (−1, 0), and topographic maps around one vertical wire. Circles are with field values of 0.333 blue, 0.5 light brown, 1.0 green, 1.5 red, 2.0 grey, 2.5 brown (A/m). In (c), The smallest circle is brown and is the closest to the electrical-current wire with a field value of 2.5 A/m. Topographic maps are superposed in (c), due to their calculations for field values and height with I/r: both in the z axe.

To highlight the resemblance between the field curves and topographic maps, one could draw a parallel between a field value of zero (A/m) and sea level on a topographic map. In Figure 2(c), a 3D representation illustrates this concept, where like in topographic maps, field curves cannot intersect with lines representing different levels [12]. The 3D rendering of curve values always maintains a planar arrangement, akin to the contour lines on a topographic map illustrating the relief contour [12].

3.2. Two Electrical-Current Wires

In this chapter, we employ two wires with identical electrical current intensities of one ampere (A) and the same direction. At any point on the planar surface, the field value is the result of combining two distinct I/r values. While field curves are

not depicted with vectors, it's worth noting that conventional magnetic field lines are not included; however, certain training courses may introduce such a classical calculation approach [13].

In Figure 3, the field curves are depicted in various colors to represent distinct physical values. The corresponding colors and their respective values can be found in Table 1. These field curve values are determined using the formula: I1/r1 + I2/r2.
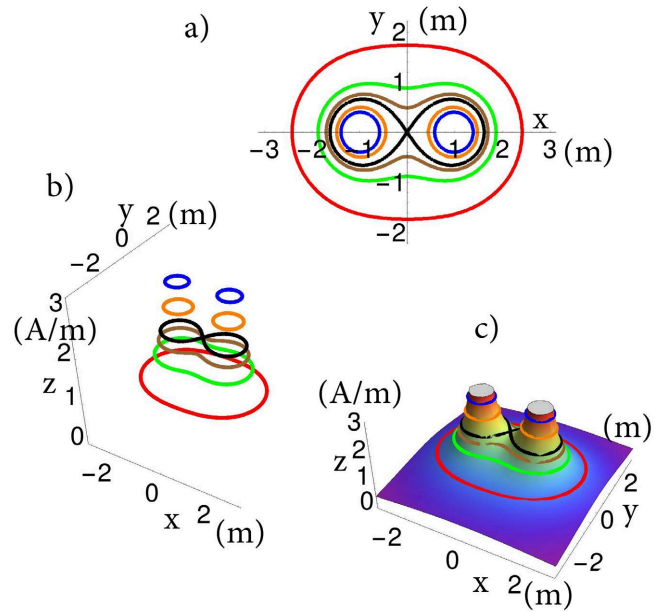

Figure 3. Three views of field curves around two parallele electrically-conducting wires with one (A) for both wires in the z direction. They are placed at xy (−1, 0) or (+1, 0). In (a), (b) and (c), blue circles are nearest to one of each electrical-current wires with a field of 3.0 (A/m). When distances are larger, the field decreases until the current field curves are around both electrical wires. In between, and as in (b) and (c), black lines forming a horizontal number 8, show a pass between the two electrical wires exactly like a pass in a topographic level map between two mountains [14]. Only the black curve cut itself ((a)-(c)).

Table 1. Around two infinite and parallel conducting wires at 1 (A/m), current field values are calculated and represented with various colors as in Figure 3.

Color lines	Red	Green	Brown	Black	Orange	Blue
Current Field value (A/m)	1.0	1.5	1.8	2.0	2.5	3.0

In Table 1, the value at the pass between the two parallel wires remains constant at 2.0 (A/m). This value persists consistently along the black curve, which resembles the infinity symbol.

3.3. Parallel and Opposite Electrical-Current Wires

In this chapter, we introduce the concept of negative field values, which appears to be employed for the first time.

The aim is to illustrate field curves around two parallel electrical-current wires with opposing intensities, as depicted in Figure 4.
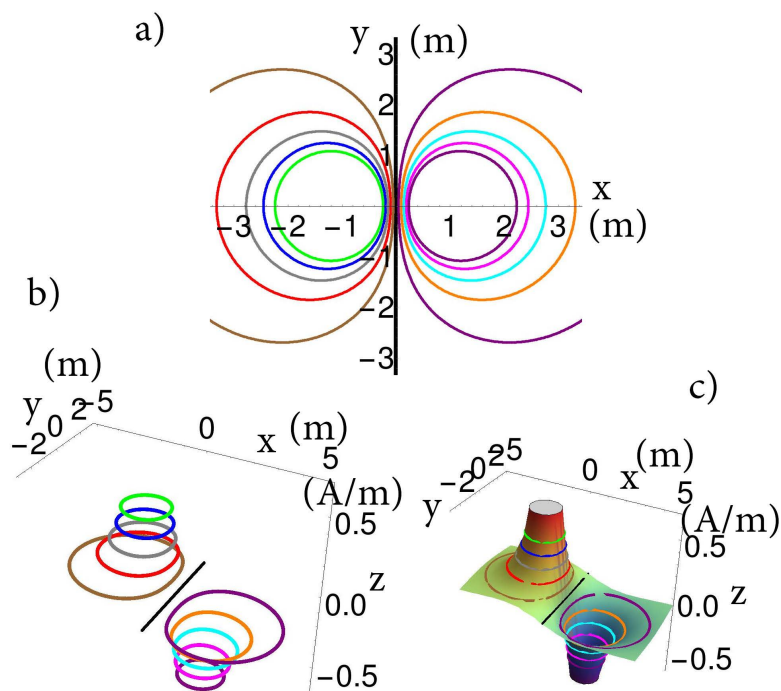
Figure 4. Two infinite wires with opposite current in z directions. Around them, field curves are at various field values. In (a), a planar view of field curves is shown at field values between +0.5 to −0.5 (A/m). In the middle, black field lines are at zero field value corresponding to the sea level of the topographic map. In (b) and (c), images are 3D views. Positive field curves above the zero-field level are in the left side, whereas in the right side they are below the sea level with negative field values.

In **Figure 4**, the wire on the left carries a positive electrical current of one ampere (A), while the wire on the right carries a negative electrical current of one ampere (A).

Also, **Figure 4** illustrates drawings of field curves at various current field values.

The black linear curve, representing sea level (*i.e.*, a field value of zero A/m), serves as a horizontal symmetrical axis. This means that a 180-degree rotation around this axis does not alter the appearance of the field curves.

When we compare the drawing in **Figure 4(a)** with the available data from M. Zollner [4], notable differences emerge. M. Zollner's work suggests that these curves take on a circular shape, whereas in **Figure 4(a)**, they do not form circles except when very close to the wires on either the left or right side.

For a comprehensive understanding of the values and colors associated with the field curves in **Figure 4**, please refer to **Table 2**.

Table 3 introduces the concept of negative field values in a scientific article for the first time. One advantage of this approach is that it facilitates comprehension by drawing parallels between field curves and the contours of a topographic map. In this analogy, positive levels correspond to elevations above sea level, while negative levels represent depths below the zero-sea level.

When considering these two opposing and parallel wires, the representation

Table 2. Relation of colored curves with field values and elevations of field lines drawn in Figure 4.

Colors	Field value	Height or depth
Green	0.5	0.5
Blue	0.4	0.4
Gray	0.3	0.3
Red	0.2	0.2
Brown	0.1	0.1
Black	0	0
Purple	−0.1	−0.1
Orange	−0.2	−0.2
Cyan	−0.3	−0.3
Maganta	−0.4	−0.4
Purple	−0.5	−0.5

Table 3. Calculation of the circle positions at level zero for (see next page) field curves at various electrical current of one (A) for the wire at position (−1, 0.) and various at position (+1, 0).

Line Colors	Right wire intensity (A)	Center (x)	Radius	Left side 0 < x < 1	Right side x > 1
Figure 5(a)					
Red	−0.1	1.02020202	0.202020202	0.818181818	1.222222222
Green	−0.2	1.083333333	0.416666667	0.666666667	1.5
Blue	−0.3	1.197802198	0.659340659	0.538461538	1.857142857
Grey	−0.4	1.380952381	0.952380952	0.428571429	2.333333333
Cyan	−0.5	1.666666667	1.333333333	0.333333333	3
Figure 5(b)					
Magenta	−0.6	2.125	1.875	0.25	4
Purple	−0.7	2.921568627	2.745098039	0.176470588	5.666666667
Brown	−0.8	4.555555556	4.444444444	0.111111111	9
Orange	−0.9	9.526315789	9.473684211	0.052631579	19
(Far)	−1	Not represented	infinity	0.0	infinity

of a field curve consistently maintains a planar structure, like the level contours found in topographic maps depicting terrain relief. Furthermore, field curves maintain a uniform field value along their entire length, mirroring the behavior of topographic contours.

3.4. Field Curves at Level Zero (A/m) in Two Parallel Electrical Wires with Opposite Current-Directions and Various Electrical-Currents.

With two distinct wires—one carrying a positive current and the other bearing various negative currents—it is possible to compute the geometry of multiple field curves.

As depicted in Figure 5, there are two wires, both with different current in ampere (A). The first wire is situated on the left side at position (−1, 0), while the second wire is on the right side at position (+1, 0). At sea level, where field values equal zero, the resulting equation is expressed as Equation (1):

$$I1/r1 + I2/r2 = 0 \text{ with } I1 > 0 \text{ and } I2 < 0 \tag{1}$$

By making mathematical adjustments of Equation (1), it yields the conventional polynomial representation of circles for the field curves, as illustrated in Figure 5.

The field curves depicted in Figure 5 exhibit a circular geometry, and the values of their field strengths and centers are provided in detail in Table 3.
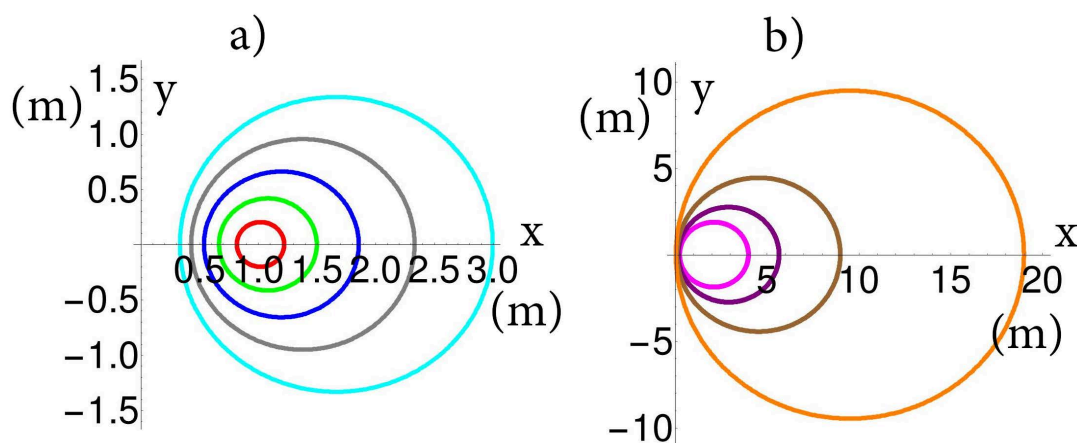

Figure 5. Sea levels with current of one (a) for the left-side electrical wire (position (−1, 0)) and various negative current for the right-side electrical wire (position (+1, 0)): negative current between −0.1 and to −0.5 A are left-side circles, and negative between −0.6 and to −0.9 A for current wire for right-side circles (see details in Table 3).

Since this paper lacks physical measurements, it is possible to express field values with precision using more than ten significant figures.

3.5. Pass Level between Different Electrical-Current (A) in Same Conducting Direction

In this section, we compare three distinct field curves. Figure 6 illustrates wires with three different currents, resulting in the formation of three distinct passes.

Under alternative electrical conditions, the values for current field passes are computed and presented in Table 4.
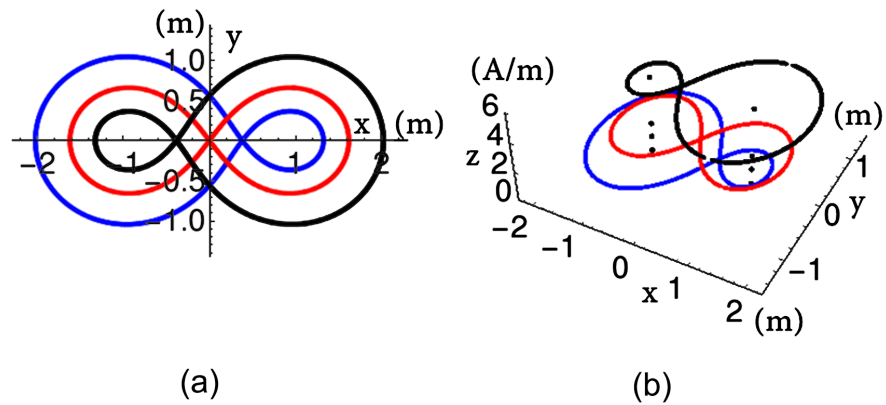
Figure 6. Planar field curves in 2D and 3D representations of pass between two electrical wires with current of 1 A in $x = -1$, and various positive currents at position $x = +1$. These representations are a superposition of three different electrical conductions. Colors are blue for right side electrical current of 0.2 A, red for 1.0 and black for 5.0. In (b), dark points indicate positions of both electrically-conducting wires.

Table 4. Calculation of current field pass values for various electrical current on the right-side and one (A) for the left-side wire.

Electrical current In (A)	Field value In (A/m)	Pass position X in (m)
0.1	0.8662277660	0.5194938
0.2	1.0472135954	0.381966
0.3	1.1977225575	0.2922212
0.4	1.3324555320	0.225148
0.5	1.4571067811	0.171573
0.6	1.5745966692	0.1270166
0.7	1.6866600265	0.08893315
0.8	1.7944271909	0.0557281
0.9	1.8986832980	0.0263340
1	2	0
1.1	2.0988088481	−0.023823
1.2	2.1954451150	−0.0455488
1.8	2.7416407865	−0.14589803
5.	5.2360697750	−0.3819660

3.6. Two Parallel Wires with Opposite Electrical Current Directions (A) and with Different Current Field Values (A/m)

This chapter elucidates the process of depicting multiple field values using two dissimilar electrical currents that flow in opposite directions. These configurations

are visualized in Figure 7, and they highlight variations in pass positions when compared to the passes between two identical electrical currents with opposing directions, as depicted in Figure 5.
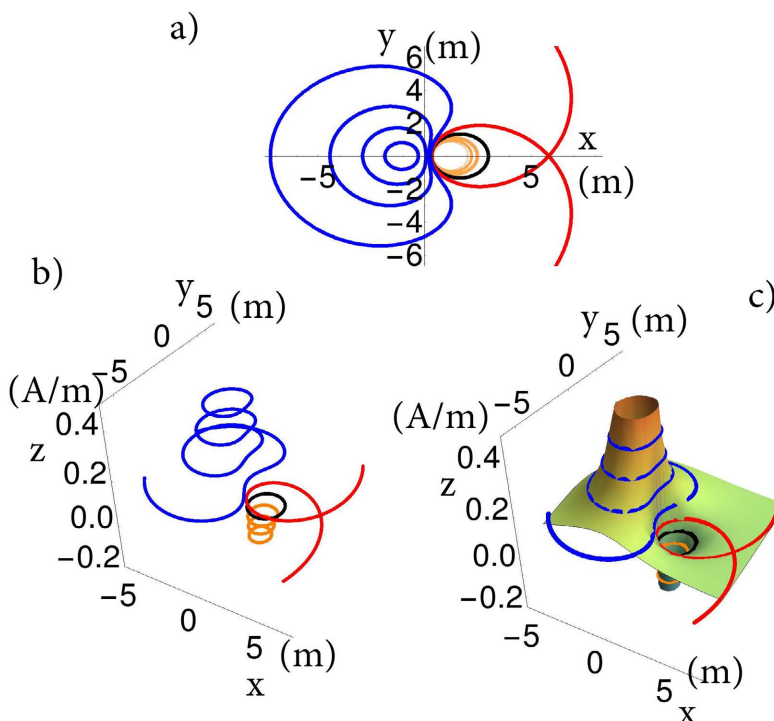

Figure 7. Planar and 3D representations of two different electrical-current wire at x = −1 (current +1 A) and x = +1 (current −0.5 A). Blue circles are at positive field values in A/m. In red, a current field values at the pass level. Black circles are at sea level with zero A/m. In orange, field values are negative. When blue curves are closer to the positive wire, there field values are higher. Symmetrically, when orange curves are closer to the negative wire, there field values are negatively further.

The field values for curves presented in Figure 7 are provided in detail in Table 5.

Table 5. Field value of field lines between two opposite different electrical currents, 1A for left-side wire and −0.5 A for right-side wire.

Colors	Field value (A/m)
Green	Up to down: 0.4 0.3 0.2 0.1 A/m
Blue	Pass field value 0.042893219 A/m
Gray	Sea level: field value at 0.0 A/m
Red	Up to down: −0.05 −0.1 −0.15 −0.2 A/m

3.7. Pass Values in 2D Level Maps between Two Opposite and Different Electrical-Currents in Parallel Wires

When considering two opposing currents with varying intensities, the pass curves

do not resemble the horizontal Figure 8 as depicted in Figure 2. Instead, these distinct configurations for six different current values are illustrated in Figure 8.
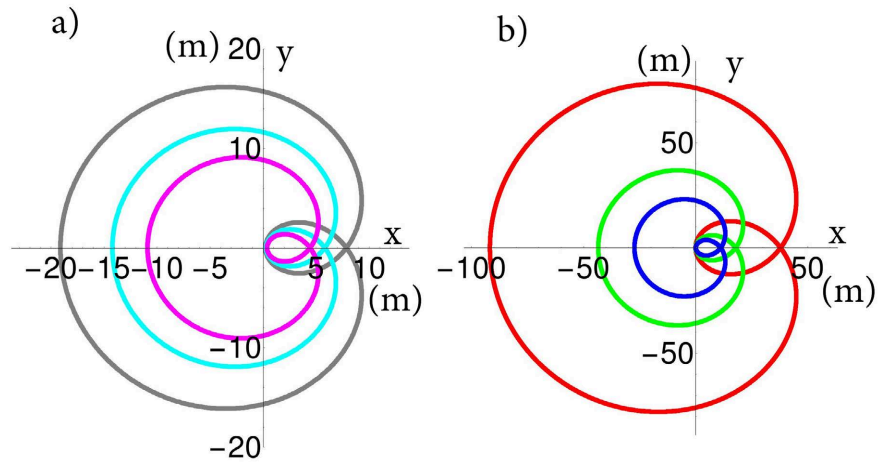

Figure 8. Various pass values in curves when wires are in opposite directions. Left current wire at 1 A, is at $x = -1$. At $x = +1$, the wire can have different negative electrical currents. In (a), field curves are at negative values: magenta at -0.9, cyan at -0.8 and grey at -0.7 A/m. In (b) drawings of field curves are at negative values: blue at -0.6, green at -0.5 and red at -0.4 A/m.

Figure 8 displays six distinct pass values corresponding to six different current values. In addition to these six values, further evaluations of pass values and their respective positions are documented in Table 6.

Table 6. Evaluations of pass positions and field values for 10 negative electrical current.

I1 (A)	I2 < 1 (A)	position x (m)	Field value at pass (A/m)	Color When drown
	−0.1	1.924945	0.233772234	
	−0.2	2.618034	0.152786405	
1	−0.3	3.422062	0.102277443	
	−0.4	4.441518	0.067544468	Magenta
	−0.5	5.828427	0.042893219	Cyan
	−0.6	7.872983	0.025403331	Grey
	−0.7	11.24441	0.013339973	Blue
	−0.8	17.94427	0.005572809	Green
	−0.9	37.97366	0.001316702	Red
	−1	Infinity	0	

4. Several Parallel Current Wires

4.1. Domains under One Electrical Wire

Domains can be established when field curves are constructed with at least one

pass. In the instance of a single electrical current wire, where no pass can be created, only one domain can be identified.

4.2. Number of Domains and Number of Pass around Parallel Wires

With several wires, domains are surrounded near and around wires by planar field curves. By adding electrical wires, the number of domains and the number of passes changes. When adding a new wire, in Table 7, the change in the number of domains and the number of passes is evaluated.

Table 7. Number of passes and domains around several parallel wires.

Number of electrical Current wires	Number of pass	Number of domains
n	n − 1	2 * n − 1

Table 7 provides the most common value for the number of passes or domains. For instance, the first domain (n = 2) includes one pass and three domains. When a third electrical current wire is introduced, a new pass is automatically generated, resulting in five domains (n = 3, following the pattern 2 * n − 1).

However, when additional wires are added, the possibility of creating more than one pass can occur. In such cases, the number of domains does not adhere to the guidelines outlined in Table 7. Consequently, when a loop traverses multiple passes, determining the number of passes and domains must be estimated manually, as described in the following chapter.

4.3. Example with Five Parallel Electrical Current Wires

In Figure 9, the arrangement is depicted where multiple passes exhibit field values of 5.34521 A/m in black and 5.73295 A/m in red.
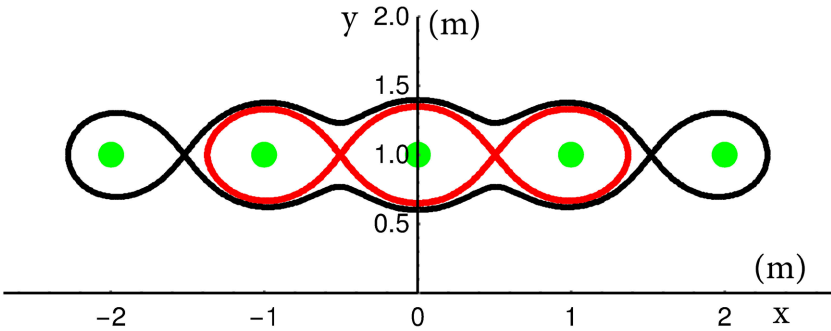

Figure 9. Pass between five identical and parallel current wires periodically placed and aligned (y = 1) with inter-distance of one (m) in x direction. Green points show positions of the five wires, and seven domains are formed. The red field curve at 5.73295 A/m has two passes as for the black field curve which is at 5.345208 A/m.

In Figure 9, the arrangement is depicted where multiple passes exhibit field values of 5.34521 A/m in black and 5.73295 A/m in red.

Also in **Figure 9**, there are five electrical current wires positioned at the center of five small green points. The electrical wire placed in the middle of these five wires is represented by the red line, which is the closest and has the highest field value of 5.73295 A/m. The black line is farther away and has a lower field value of 5.345208 A/m. As anticipated, these current field curves do not intersect. Surprisingly, the number of domains observed is seven, which is fewer than the expected number of nine based on the direct calculation from **Table 7**.

5. Discussion

5.1. Gauss's Law of Magnetism

In Gauss's law of magnetism, it is asserted that magnetic field lines neither have a distinct starting point nor an endpoint; they extend infinitely [15]. Every 3D illustration in this current article adheres to this principle. These field lines are nearly circular, with only one extending to infinity, as seen in **Figure 4**.

In our model, each curve maintains a uniform field value throughout its length. Therefore, when certain positions of the curves are at an infinite distance from the wires, all positions must exhibit a field value of zero (A/m).

This characteristic, exemplified by the linear black curve in **Figure 4**, represents an enhancement of Gauss's law by introducing the concept of zero field value in straight lines.

5.2. Circle Field Curves

M. Zollner conducted experiments involving magnetic fields with two parallel current wires. When the currents were in opposite directions, he represented the field lines as eccentric circles [4]. In **Figure 4** of this article, the field lines may indeed resemble circles when they are near one of the current wires. However, as the field values approach zero, the lines progressively adopt a more linear configuration around the midpoint between the two conducting wires. Through this comparison, it becomes evident that both theoretical and experimental lines are not significantly discrepant.

One notable advantage of the theory proposed in this article is its applicability to highly intricate electrical scenarios, offering a valuable tool for analyzing complex electrical configurations.

5.3. Superposition Principle for Field Curves

Regarding magnetic field lines, Feynman remarked that "The field lines, however, are only a crude way of describing a field, and it is very difficult to give the correct, quantitative laws directly in terms of field lines". He also noted that "the ideas of the field lines do not contain the deepest principle of electrodynamics, which is the superposition principle", Furthermore, he pointed out that "we don't get any idea about what the field line patterns will look like when both sets are present together" [16].

In our paper, we introduce a novel paradigm for field values, employing a dis-

tinct theoretical approach to construct curves based on electrical current fields. In Figure 1(b), the addition of field values (without vectors) enables the representation of highly intricate structures. As a result, it appears that our article adheres to the superposition principle, as complex field configurations can be effectively delineated using this approach.

At this juncture in the article, the reader might find that a clear and comprehensible presentation has been provided, substantiating the author's claims effectively.

6. Conclusion

This article does not use the conventional magnetic field vectors to represent field curves and instead employs specific field values for each point surrounding electrical wires. The field value is locally equal to the ratio of current divided by distance (I/r). In the case of parallel wires, these points are arranged into planar curves, which can assume various forms that are easily calculated through a linear summation of the field values. These unique characteristics bear a resemblance to the level curves found in topographic maps. Thanks to this analogy, a current field curve can intersect itself through a pass, but only once for each conducting wire. Looking ahead there is potential for extending our previous model to encompass non-parallel wire configurations, which could present intriguing opportunities for further exploration. The main application of our proposed model will probably be a precise drawing of field lines in electrical motors.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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