Asian Journal of Probability and Statistics

Volume 26, Issue 4, Page 1-7, 2024; Article no.AJPAS.115262

ISSN: 2582-0230

A Note on the Risk Model with Dependence and Capital Injections

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Author's contribution

The sole author designed, analyzed, interpreted and prepared the manuscript.

Article Information

DOI: 10.9734/AJPAS/2024/v26i4604

Open Peer Review History:

This journal follows the Advanced Open Peer Review policy. Identity of the Reviewers, Editor(s) and additional Reviewers, peer review comments, different versions of the manuscript, comments of the editors, etc are available here: https://www.sdiarticle5.com/review-history/115262

Received: 22/01/2024

Accepted: 27/03/2024

Original Research Article

Published: 08/04/2024

Abstract

The note considers a risk model with dependence and capital injections, where the dependence structure is modeled by a Farlie-Gumbel-Morgenstern copula. In the risk model, the initial surplus starts from a level $u \geq h$, where $h > 0$ is a fix constant. The author derives an expression for the Laplace transform of the Gerber-Shiu function. In particular, an explicit formula for the Gerber-Shiu function is obtained when the initial surplus is h.

Keywords: Gerber-Shiu function; dependence; capital injections; laplace transform.

1 Introduction

Nie et al. [1,2] introduced a risk model with capital injections. In the model the insurer's initial surplus starts from a level $u \geq h$,($h > 0$ is a fixed constant). In any case, if the surplus drops from above h to between 0 and h , the capital injection will restore the surplus level to h. If the surplus drops from a level above$h$ to below 0, the

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ruin will occur. Dickson, D. C., Qazvini, M. [3], studied a risk model with capital injections and obtained an expression for its Laplace transform and Gerber-Shiu function itself when the initial surplus is h, providing an effective method for studing ruin related quantities in finite time. Zhao et al. [4] studied the optimal periodic dividend and capital injection problem for the spectral positive Lévy processes, and maximized the total value of the expected discounted dividend and penalty discounted capital injections until the time of ruin. They found that the optimal return function can be expressed as a scaling function. Xu et al. [5] considered a capital injection strategy based on the periodic implementation of claims in the classical Poisson risk model, and derived an explicit expression for the discount density of surplus levels after a certain number of claims before the ruin occurred. They also found an explicit Laplace transform expression for the time of ruin. Yu et al. [6] considered a classical risk model with a periodic capital injection strategy and a barrier dividend strategy, and derived boundary conditions for the Gerber-Shiu function, the expected discounted capital injection function and the expected discounted dividend function by assuming that the observation interval and claim amount are exponentially distributed, respectively.

In ruin theory, the classical compound Poisson risk model with independence between the claim size and interclaim time. In the study of the classical compound Poisson risk model, it is assumed that the claim sizes and the interclaim times are mutually independent. Although this hypothesis is applicable to certain practical situations and simplifies the study of the calculation of the amount of ruin of interest, it has been proved to be inappropriate and restrictive in other practical contexts. For example, in modeling damages due to natural catastrophic events the intensity of the catastrophe and the time elapsed,becuase the last catastrophe are expected to be dependent. See e.g. Boudreault [7] and Nikoloulopoulos, A. K., Karlis, D. [8] for an application of this type of dependence structure in an earthquake contex. Recently, many authors have paid lots of attention to the risk model with dependence between interclaim times and claim sizes. Cossette et al. [10], considered the classical compound Poisson risk model with dependence structure based on a Farlie-Gumbel-Morgennstern(FGM) copula, and evaluated the defective renewal equation for the Gerber-Shiu function. Zhang, Z., Yang, H. [10] studied the Gerber-Shiu function for a perturbed by diffusiom compound Poisson risk model with dependence structure between the claim size and interclaim time by FGM copula, and show that the Gerber-Shiu function satisfy some defective renewal equations. Shi et al. [11] consider the compound Poisson risk model with a threshold dividend strategy and dependence structure modeled by a FGM copula, and derive explicit formulas for Gerber-shiu functions and expected discounted divided payments.

In the note, we study risk model with capital injections and dependence between the claim sizes and interclaim times, based on a FGM copula. We derive an expression for the Laplace transform of the Gerber-Shiu function, and the corresponding result in [3] is generalized by this note.

2 Model Description

Consider insurer's surplus process at time t defined as $\{U(t), t \geq 0\}$, with

$$U(t) = u + pt - \sum_{i=1}^{N(t)} Z_i \tag{2.1}$$

where u is the initial surplus, $p > 0$ is the premium rate which is asuumed to be a positive constant.$\sum_{i=1}^{N(t)} Z_i$ is a compound Poisson process and is the total claim amount process. $\{N(t)\}_{t\geq 0}$ is a Poisson process with Poisson parameter λ and $\{Z_i\}_{i=1}^{\infty}$ are assumed to form a sequence of independent identically distributed(i.i.d.) random variables(r.v.), where Z_i represents the amount of the ith claim. Let $g(x)$ be the probability density function of Z_i, and the cumulative distribution function(c.d.f.) of Z_i is $G(x)$, with $G(0) = 0$, and $G = 1 - \bar{G}$. The interclaim times $\{V_i, i \geq 1\}$ with V_i the time between the $(i-1)th$ and the ith claim, are i.i.d with common p.d.f. $p(t) = \lambda e^{-\lambda t}$. Obviously $(Z_i, V_i), i \geq 1$, are i.i.d. random vectors. Motivated by Cossette et al. [9] and Denuit et al. [12], we use the FGM copula to define the joint distribution of (Z, V) and the claim size and the interclaim time is dependet. The FGM copula is given by

$$C_\theta^{FGM}(u_1, u_2) = u_1 u_2 + \theta u_1 u_2 (1 - u_1)(1 - u_2)$$

$\theta \in [-1,1]$, $u_1, u_2 \in [0,1]$, we assume that, the joint p.d.f. of $\{(Z_i, V_i), i \geq 1\}$ is defined by

$$g(x,t) = g(x)\lambda e^{-\lambda t} + \theta l(x)(2\lambda e^{-2\lambda t} - \lambda e^{-\lambda t})$$

$x, t \geq 0$, where $l(x) = (1-2G(x))g(x)$, $x \geq 0$ with $\hat{l}(s)$ is the Laplace transform of $l(x)$.

Let $\tau = \inf\{t : U(t) < 0 | U(0) = u\}$ be the time of ruin with $\tau = \infty$ if $U(t) > 0$ for all $t > 0$(i.e. tuin does not occur)and $\varphi(u) = Pr(T < \infty | U(0) = u)$ be the ultimate ruin probability. To guarantee that ruin will not occur, the premium loading factor is given by $p > \lambda E(Z_1)$.

The note is interested in the risk model with dependence and capital injection. Further, the initial surplus starts from $u \geq h > 0$, when the surplus process falls between 0 and h, a capital injection causes the surplus to start from k. When the surplus falls below 0, ruin occurs. Therefor, we use the same notation as for the classical risk model, with a subscript h.

3 Analysis of the Gerber-Shiu Function

In this section, we will investigate the Gerber-Shiu function of the risk process. In Dickson et al. [3], the authors introduce Gerber-Shiu function about τ_h, N_{τ_h}, and the penalty function $\omega(x,y)$,when $x \geq h$ and $y > 0$, as follows

$$m(u) = E\left[r^{N_{\tau_h}} exp(-\delta\tau_h)\omega(U(\tau_h^-), |U(\tau_h)|)I(\tau_h < \infty)|U(0) = u\right] \tag{3.1}$$

where $\delta \geq 0$, $0 < r \leq 1$ as well as $u > h$, $U(\tau_h^-)$ is the surplus prior to ruin. In Landriault et al. [13], authors explain δ as the parameter of a Laplace transform and r as the parameter of a probability generating function. Expecially, $m(u) = 0$, when $0 \leq u < h$.

We need to introduce the well-known Dickon-Hipp operator [14] T_s of a real-valued integrable function g defined as

$$T_s g(x) = \int_x^\infty e^{-s(y-x)} g(y) \mathrm{d}y$$

Theorem 1.The Gerber-Shiu penalty function $m(u)$ satisfies

$$m(h) = \frac{\frac{r\lambda}{p}[(\rho_2 - \frac{2\lambda+\delta}{p})T_{\rho_2}\beta_1(h) - (\rho_1 - \frac{2\lambda+\delta}{p})T_{\rho_1}\beta_1(h)] + \frac{r\lambda\theta}{p}[(\rho_2 - \frac{\delta}{p})T_{\rho_2}\beta_2(h) - (\rho_1 - \frac{\delta}{p})T_{\rho_1}\beta_2(h)]}{(\rho_2 - \rho_1) - \frac{r\lambda}{p}[(\rho_2 - \frac{2\lambda+\delta}{p})\eta_2(h) - (\rho_1 - \frac{2\lambda+\delta}{p})\eta_1(h)] - \frac{r\lambda\theta}{p}[(\rho_2 - \frac{\delta}{p})\xi_2(h) - (\rho_1 - \frac{\delta}{p})\xi_1(h)]} \tag{3.2}$$

where,

$$\gamma_1(u) = \int_0^{u-h} m(u-x)g(x)\mathrm{d}x + \int_{u-h}^{u} m(h)g(x)\mathrm{d}x + \beta_1(u)$$

$$\beta_1(u) = \int_u^\infty \omega(u, x-u)g(x)\mathrm{d}x$$

$$\gamma_2(u) = \int_0^{u-h} m(u-x)l(x)\mathrm{d}x + \int_{u-h}^{u} m(h)l(x)\mathrm{d}x + \beta_2(u)$$

$$\beta_2(u) = \int_u^\infty \omega(u, x-u)l(x)\mathrm{d}x$$

$$\eta_1(h) = \int_h^\infty exp(-\rho_1(u-h))(\bar{G}(u-h) - \bar{G}(u))\mathrm{d}u$$

$$\eta_2(h) = \int_h^\infty exp(-\rho_2(u-h))(\bar{G}(u-h) - \bar{G}(u))\mathrm{d}u$$

$$\xi_1(h) = \int_h^\infty exp(-\rho_1(u-h))(\bar{L}(u-h) - \bar{L}(u))\mathrm{d}u$$

$$\xi_2(h) = \int_h^\infty exp(-\rho_2(u-h))(\bar{L}(u-h) - \bar{L}(u))\mathrm{d}u$$

Proof. By conditioning on the time and amount of the first claim we can obtain for $u \geq h$,

$$\begin{aligned}
m(h) =& E[r^{N_t} exp(-\delta t) m(u+pt-x) | X_1 = x, V_1 = t] + E[r^{N_t} exp(-\delta t) m(h) | X_1 = x, V_1 = t] \\
&+ E[[r^{N_t} exp(-\delta T_h) \omega(U(T_h^-), |U(T_h)|) I(T_h < \infty) | U(0) = u] | X_1 = x, V_1 = t] \\
&= \int_0^\infty \left[\int_0^{u+pt-h} exp(-\delta t) r m(u+pt-x) g(x,t) \mathrm{d}x + \int_{u+pt-h}^{u+pt} exp(-\delta t) r m(h) g(x,t) \mathrm{d}x \right. \\
&+ \left. \int_{u+pt}^\infty r exp(-\delta t) \omega(U(T_h^-), |U(T_h)|) g(x,t) \mathrm{d}x \right] \mathrm{d}t \\
&= \frac{r\lambda}{p} \int_u^\infty exp(-(\lambda+\delta) \frac{\tau-u}{p}) \left[\int_0^{\tau-h} m(\tau-x) g(x) \mathrm{d}x + \int_{\tau-h}^{\tau} m(h) g(x) \mathrm{d}x + \int_\tau^\infty f(x) \omega(\tau, x-\tau) \mathrm{d}x \right] \mathrm{d}\tau \\
&+ \frac{r\lambda\theta}{p} \int_u^\infty (2exp(-(2\lambda+\delta)\frac{\tau-u}{p}) - exp(-(\lambda+\delta)\frac{\tau-u}{p})) \left[\int_0^{\tau-h} m(\tau-x) l(x) \mathrm{d}x + \int_{\tau-h}^{\tau} m(h) l(x) \mathrm{d}x \right. \\
&+ \left. \int_\tau^\infty l(x) \omega(\tau, x-\tau) \mathrm{d}x \right] \mathrm{d}\tau \\
&= \frac{r\lambda}{p} \int_u^\infty exp(-(\lambda+\delta)\frac{\tau-u}{p}) \gamma_1(\tau) \mathrm{d}\tau + \frac{r\lambda\theta}{p} \int_u^\infty (2exp-(2\lambda+\delta)\frac{\tau-u}{p}) - exp(-(\lambda+\delta)\frac{\tau-u}{p})) \gamma_2(\tau) \mathrm{d}\tau
\end{aligned} \tag{3.3}$$

Using the operator T_s we can obtain

$$m(u) = \frac{r\lambda}{p} T_{\frac{\lambda+\delta}{p}} \gamma_1(u) + \frac{r\lambda\theta}{p} \left(2T_{\frac{2\lambda+\delta}{p}} \gamma_2(u) - T_{\frac{\lambda+\delta}{p}} \gamma_2(u) \right) \tag{3.4}$$

Noting $m(u) = 0$, when $0 \leq u < h$, then

$$T_s m(h) = \int_h^\infty exp(-s(x-h)) m(x) \mathrm{d}x = e^{sh} \hat{m}(s)$$

therefore, $T_s \gamma_1(h) = e^{sh} \hat{\gamma}_1(s)$, $T_s \gamma_2(h) = e^{sh} \hat{\gamma}_2(s)$. By the Dickson-Hipp operator, we have

$$\begin{aligned}
T_s m(h) =& \frac{r\lambda}{p} T_s T_{\frac{\lambda+\delta}{p}} \gamma_1(h) + \frac{r\lambda\theta}{p} (2T_s T_{\frac{2\lambda+\delta}{p}} \gamma_2(h) - T_s T_{\frac{\lambda+\delta}{p}} \gamma_2(p)) \\
=& \frac{r\lambda}{p} \times \frac{T_{\frac{\lambda+\delta}{p}} - T_s}{(s - \frac{\lambda+\delta}{p})} \gamma_1(h) + \frac{r\lambda\theta}{p} \left(2 \times \frac{T_{\frac{2\lambda+\delta}{p}} - T_s}{(s - \frac{2\lambda+\delta}{p})} \gamma_2(h) - \frac{T_{\frac{\lambda+\delta}{p}} - T_s}{(s - \frac{\lambda+\delta}{p})} \gamma_2(h) \right)
\end{aligned} \tag{3.5}$$

Multiplying (3.5) by $(s - \frac{\lambda+\delta}{p})(s - \frac{2\lambda+\delta}{p})$, we have

$$\begin{aligned}
(s - \frac{\lambda+\delta}{p})(s - \frac{2\lambda+\delta}{p}) T_s m_{r,\delta}(h) =& \frac{r\lambda}{p} (s - \frac{2\lambda+\delta}{p}) (T_{\frac{\lambda+\delta}{p}} - T_s) \gamma_1(h) \\
&+ 2\frac{r\lambda\theta}{p} (s - \frac{\lambda+\delta}{p}) (T_{\frac{2\lambda+\delta}{p}} - T_s) \gamma_2(h) \\
&- \frac{r\lambda\theta}{p} (s - \frac{2\lambda+\delta}{p}) (T_{\frac{\lambda+\delta}{p}} - T_s) \gamma_2(h)
\end{aligned} \tag{3.6}$$

Multiplying (3.4) by $(s - \frac{\lambda+\delta}{p})$ and rewrite (3.5), we can obtain

$$\begin{aligned}
(s - \frac{\lambda+\delta}{p})(s - \frac{2\lambda+\delta}{p}) T_s m(h) =& (s - \frac{2\lambda+\delta}{p}) m(k) + \frac{\lambda}{p} \times 2\frac{r\lambda\theta}{p} T_{\frac{2\lambda+\delta}{p}} \gamma_2(h) \\
&- \frac{r\lambda}{p} (s - \frac{2\lambda+\delta}{p}) T_s \gamma_1(h) - \frac{r\lambda\theta}{p} (s - \frac{\delta}{p}) T_s \gamma_2(h)
\end{aligned} \tag{3.7}$$

$$
\begin{aligned}
T_s\gamma_1(h) =& \int_h^\infty exp(-s(u-h))\gamma_1(u)\mathrm{d}u \\
&= exp(sh)\int_h^\infty exp(-su)\int_0^{u-h} g(x)m(u-x)\mathrm{d}x\mathrm{d}u + m(h)\int_h^\infty exp(-s(u-h))\int_{u-h}^u g(x)\mathrm{d}x\mathrm{d}u \\
&+ \int_h^\infty exp(-s(u-h))\beta_1(u)\mathrm{d}u \\
&= exp(sh)\hat{g}(s)\hat{m}(s) + m(h)\int_h^\infty exp(-s(u-h))(\bar{G}(u-h)-\bar{G}(u))\mathrm{d}u + T_s\beta_1(h)
\end{aligned} \tag{3.8}
$$

Since $T_s m(h) = e^{sh}\hat{m}(s)$, we have

$$
T_s\gamma_1(h) = T_s m(h)\hat{g}(s) + m(h)\int_h^\infty exp(-s(u-h))(\bar{G}(u-h)-\bar{G}(u))\mathrm{d}u + T_s\beta_1(h)
$$

Similaly

$$
T_s\gamma_2(h) = T_s m(h)\hat{l}(s) + m(h)\int_h^\infty exp(-s(u-h))(\bar{L}(u-h)-\bar{L}(u))\mathrm{d}u + T_s\beta_2(h)
$$

Substiituting in (3.7), we can derive

$$
\begin{aligned}
(s-\frac{\lambda+\delta}{p})(s-\frac{2\lambda+\delta}{p})T_s m(h) =&(s-\frac{2\lambda+\delta}{p})m(h) - \frac{r\lambda}{p}(s-\frac{2\lambda+\delta}{p})\left[T_s m(h)\hat{g}(s)\right. \\
&\left.+m(h)\int_h^\infty exp(-s(u-h))(\bar{G}(u-h)-\bar{G}(u))\mathrm{d}u + T_s\beta_1(h)\right] \\
&-\frac{r\lambda\theta}{p}(s-\frac{\delta}{p})\left[T_s m(h)\hat{l}(s)\right. \\
&\left.+m(h)\int_h^\infty exp(-s(u-h))(\bar{L}(u-h)-\bar{L}(u))\mathrm{d}u + T_s\beta_2(h)\right] \\
&+\frac{\lambda}{p}\times\frac{2r\lambda\theta}{p}\times e^{\frac{2\lambda+\delta}{p}}\hat{\gamma}_2(s)
\end{aligned} \tag{3.9}
$$

$$
\begin{aligned}
T_s m(h) =&\frac{1}{(s-\frac{\lambda+\delta}{p})(s-\frac{2\lambda+\delta}{p})+\frac{r\lambda}{p}(s-\frac{2\lambda+\delta}{p})\hat{g}(s)+\frac{r\lambda\theta}{p}(s-\frac{\delta}{p})\hat{l}(s)} \\
&\times\left\{(s-\frac{2\lambda+\delta}{p})m(h) - \frac{r\lambda}{p}(s-\frac{2\lambda+\delta}{p})\left[m(h)\int_h^\infty exp(-s(u-h))(\bar{G}(u-h)-\bar{G}(u))\mathrm{d}u + T_s\beta_1(h)\right]\right. \\
&-\frac{r\lambda\theta}{p}(s-\frac{\delta}{p})\left[m(h)\int_h^\infty exp(-s(u-h))(\bar{L}(u-h)-\bar{L}(u))\mathrm{d}u + T_s\beta_2(h)\right] \\
&\left.+\frac{\lambda}{p}\times\frac{2r\lambda\theta}{p}\times T_{\frac{2\lambda+\delta}{p}}\gamma_2(h)\right\}
\end{aligned} \tag{3.10}
$$

Since $T_s m(h) = e^{sh}\hat{m}(s)$, we can rewrite (3.10)

$$
\begin{aligned}
\hat{m}(s) =&\frac{1}{(s-\frac{\lambda+\delta}{p})(s-\frac{2\lambda+\delta}{p})+\frac{r\lambda}{p}(s-\frac{2\lambda+\delta}{p})\hat{g}(s)+\frac{r\lambda\theta}{p}(s-\frac{\delta}{p})\hat{l}(s)} \\
&\times\left\{(s-\frac{2\lambda+\delta}{p})exp(-sh)m(h) - \frac{r\lambda}{p}(s-\frac{2\lambda+\delta}{p})\left[m(h)\int_h^\infty exp(-su)(\bar{G}(u-h)-\bar{G}(u))\mathrm{d}u + exp(-sh)T_s\beta_1(h)\right]\right. \\
&-\frac{r\lambda\theta}{p}(s-\frac{\delta}{p})\left[m(h)\int_h^\infty exp(-su)(\bar{L}(u-h)-\bar{L}(u))\mathrm{d}u + exp(-sh)T_s\beta_2(h)\right] \\
&\left.+\frac{\lambda}{p}\times\frac{2r\lambda\theta}{p}\times exp(-sh)T_{\frac{2\lambda+\delta}{p}}\gamma_2(h)\right\}
\end{aligned} \tag{3.11}
$$

where, the denominator of Eq. (3.11) $(s-\frac{\lambda+\delta}{p})(s-\frac{2\lambda+\delta}{p})+\frac{r\lambda}{p}(s-\frac{2\lambda+\delta}{p})\hat{f}(s)+\frac{r\lambda\theta}{p}(s-\frac{\delta}{p})\hat{h}(s)=0$ is the Lundberg's generalized equation with two different positive real roots $\rho_i,\ i=1,2$. See the analysis of the Lunderberg's generalized equation [10] and [11]. These roots must also be roots of the numerator of Eq. (3.11), given that it is analytic. Therefore we obtain $m(k)$ by the following linear systerm.

$$m(h)=\frac{\left\{\frac{r\lambda}{p}(\rho_1-\frac{2\lambda+\delta}{p})T_{\rho_1}\beta_1(h)+\frac{r\lambda\theta}{p}(\rho_1-\frac{\delta}{p})T_{\rho_1}\beta_2(h)-\frac{\lambda}{p}\times\frac{2r\lambda\theta}{p}\times T_{\frac{2\lambda+\delta}{p}}\gamma_2(h)\right\}}{\left\{(\rho_1-\frac{2\lambda+\delta}{p})-\frac{r\lambda}{p}(\rho_1-\frac{2\lambda+\delta}{p})\eta_1(h)-\frac{r\lambda\theta}{p}(\rho_1-\frac{\delta}{p})\xi_1(h)\right\}} \tag{3.12}$$

$$m(h)=\frac{\left\{\frac{r\lambda}{p}(\rho_2-\frac{2\lambda+\delta}{p})T_{\rho_2}\beta_1(h)+\frac{r\lambda\theta}{p}(\rho_1-\frac{\delta}{p})T_{\rho_2}\beta_2(h)-\frac{\lambda}{p}\times\frac{2r\lambda\theta}{p}\times T_{\frac{2\lambda+\delta}{p}}\gamma_2(h)\right\}}{\left\{(\rho_2-\frac{2\lambda+\delta}{p})-\frac{r\lambda}{p}(\rho_2-\frac{2\lambda+\delta}{p})\eta_2(h)-\frac{r\lambda\theta}{p}(\rho_2-\frac{\delta}{p})\xi_2(h)\right\}} \tag{3.13}$$

Hence,

$$m(h)=\frac{\frac{r\lambda}{p}[(\rho_2-\frac{2\lambda+\delta}{p})T_{\rho_2}\beta_1(h)-(\rho_1-\frac{2\lambda+\delta}{c})T_{\rho_1}\beta_1(h)]+\frac{r\lambda\theta}{p}[(\rho_2-\frac{\delta}{p})T_{\rho_2}\beta_2(h)-(\rho_1-\frac{\delta}{p})T_{\rho_1}\beta_2(h)]}{(\rho_2-\rho_1)-\frac{r\lambda}{p}[(\rho_2-\frac{2\lambda+\delta}{p})\eta_2(h)-(\rho_1-\frac{2\lambda+\delta}{c})\eta_1(h)]-\frac{r\lambda\theta}{p}[(\rho_2-\frac{\delta}{p})\xi_2(h)-(\rho_1-\frac{\delta}{c})\xi_1(h)]} \tag{3.14}$$

The proof is completed. □

When $\theta=0$, the claim amount r.v. X_j and the interclaim time r.v. W_j is independent,

$$g(x,t)=g(x)\lambda e^{-\lambda t}$$

then,

$$m(h)=\frac{\frac{r\lambda}{p}\int_h^\infty\int_u^\infty e^{-\rho(u-h)}g(x)\omega(u,x-u)\mathrm{d}x\mathrm{d}u}{1-\frac{r\lambda}{p}\int_h^\infty e^{-\rho(u-h)}(\bar{G}(u-h)-\bar{G}(u))\mathrm{d}u} \tag{3.15}$$

Which coincides with the result obtained by Dickson et al. [3].

4 Conclusions

a In the section (2), a risk model with capital injections and a dependence structure modeled by a Farlie-Gumbel-Morgenstern copula is described.

b In the section (3), an explicit formula for the Gerber-Shiu function is obtained when the initial surplus is k.

Acknowledgment

The author is grateful to the anonymous referee for constructive comments which improved the quality of this note.

Competing Interests

Author has declared that no competing interests exist.

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